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AFCRC-TN-60-231

DEVELOPMENT OF A METHOD OF FORECASTING THE OCCURRENCE AND BASE ALTITUDE OF CONTRAIL FORMATION

Weather Services, Inc.
200 Berkeley Street
Boston, Mass.

Scientific Report

AF 19 (604) - 1961

January 20, 1960

Prepared
for

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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INTRODUCTION

Originally, the primary object of this project was the study of contrail formation for altitudes above fifty thousand feet, directed toward a further knowledge of contrail formation. More accurate forecasts could then be made. Also with a better understanding of the physics of contrails a method of suppression or eventual elimination of contrails could be developed.

The basic data for this study was to be obtained from firings at Cape Canaveral, Florida. It proved to be virtually impossible to obtain sufficient data upon which to base a study. While awaiting this data it was found that, although a method of forecasting formation of contrails below fifty thousand feet had been developed, there was still much work to be done in this field. Also, a need did exist for a method of forecasting the height at which contrails formed. Attention was then focused on these two problems.

During the period 1 December, 1954 to 15 December, 1955 extensive contrail data was collected by the U.S.A.F. under Air Force "Project Cloud Trail". This data was made available to Weather Services as a basis for the further study of the subject of contrail formation and altitude at which these contrails develop.

ABSTRACT

To forecast the occurrence of contrails, a simple mathematical predicting model was developed. This was based primarily on the shape of the ascent curve. The mathematical problem was one of discriminant analysis, in which a linear combination of three slopes was found, which maximized the ratio of the variability of the resulting combination between the two categories, contrails and no contrails. The discriminant values found for all cases separated well ranging from 85% to 100% correct.

In the forecasting of the height of contrail formation, an equation was developed using the prevailing height of the tropopause to indicate approximat height of contrail formation and the height at which the -50C t nperature occurred. The data was normalized providing a forecasting method, effective regardless of location. A graph of this equation resulted in a simple method of forecasting contrail height.

DATA USED

The data used resulted from Air Force "Project Cloud Trail". This study was published in February 1956 in an AWS Manual (AWS TR 105-132). The data was collected from 1, December, 1954 to 15, December, 1955, in the vicinity of twenty-one upper air sounding stations.

The procedure used in the observational phase of "Project Cloud Trail" was as follows:

a. Daily at approximately one hour before or two hours after 1530Z, two aircraft were vectored to point 25,000 feet above an upper-air sounding station. The aircraft then climbed to the maximum altitude obtainable, maintaining position within 30 miles of the station.

b. The wingman observed whether or not the lead aircraft produced exhaust trails, noting if they were continuous or intermittent, weak or strong, including basis and tops of layers in which the trails formed.

In this study observational data for Caribou, Maine and Sault St. Marie, Michigan was insufficient to warrant their inclusion.

DATA LIMITATION

During the period of data collection, the maximum altitude of the aircraft seldom exceeded 45,000 feet. Therefore, this study was restricted to data observed below the height. Using this forecasting technique on data for altitude above 45,000 feet, should be accomplished with this restriction in mind. This in no way implies that the "altitude formation forecast method" developed herein would not be valid at higher levels, but merely that its validity has not been tested.

FORECASTING THE OCCURRENCE OF CONTRAIL FORMATION

METHOD

Contrail data was examined with the corresponding soundings endeavoring to categorize the conditions necessary for formation. Such variables as tropopause height and lapse rate proved to be related to the formation, therefore a method of describing the general shape of the temperature curve was needed.

In order to completely describe any temperature ascent curve, it would be necessary to use an infinite number of points. However, in this case, only that portion of the ascent associated with the existence of contrails required description. This was accomplished by selecting values at three set points. Generally speaking, the slopes of the curve at -45°C, -50°C and -55°C were used to characterize the behavior of the curve although, in order to obtain the best results, the three temperatures used, differed slightly from station to station. These slopes were computed on the basis of the temperature change (ΔT) over a range measured from one thousand feet above and below the three temperatures selected. (see table 1) Thus three variables as slope measurements were to characterize the general behavior of the ascent curve in the vicinity of the contrail occurrence.

The mathematical problem was one of discriminant analysis, in which a linear combination of the three slopes was found, so as to maximize the ratio of the variability of the resulting combination between the two categories, contrails and no contrails, as compared with the variation within the given category. A linear combination of the measurements was determined which attempted to place the resulting number in the correct category.

In this discriminant analysis, three variables were associated with each case of contrails and a similar three variables with each case of no contrails; the three variables being the slope of the ascent at -45, -50 and -55°C. The discriminant process is similar to a regression analysis in which three are only two categories, as in this case namely contrails and no contrails.

TABLE I

TEMPERATURES VARIABLES FOR PITTSBURG

CATEGORY I (CONTRAILS)			CATEGORY II (NO CONTRAILS)		
-45°C	-50°C	-55°C	-45°C	-50°C	-55°C
-5.6	-5.3	-5.8	-4.4	-2.0	0
-4.5	-4.0	-3.5	-3.1	-1.6	0
-4.4	-4.1	-3.0	-2.2	-2.6	-1.5
-5.3	-4.3	-3.8	-4.2	-4.1	-2.9
-6.4	-5.0	-4.5	-3.6	-0.7	0
-5.2	-4.9	-4.5	-3.3	-1.5	0
-6.0	-5.6	-4.4	-3.8	0	0
-5.0	-4.9	-4.0	-5.0	0	0
-5.9	-5.0	-5.4	-3.5	-1.8	0
-5.2	-5.0	-4.5	-4.0	-4.2	-1.6
-4.8	-5.0	-4.5	-4.8	-5.1	0
-5.0	-5.0	-5.5	-4.4	-1.8	-1.8
-4.8	-5.3	-4.7	-2.4	-1.8	-3.3
-5.1	-5.5	-5.0	-0.6	-3.0	-3.1
-5.5	-5.0	-4.8	-3.5	-0.4	0
-4.6	-5.0	-5.6	-2.7	-1.4	0
-5.1	-5.4	-5.1	-3.1	-0.4	-2.5
-5.0	-4.7	-4.9			
-4.5	-5.0	-1.1			
-4.7	-4.0	-3.4			
-4.7	-5.0	-5.1			
-5.0	-4.9	-4.1			
-6.0	-5.7	-6.0			
-3.6	-4.0	-3.5			
-4.7	-5.0	-4.5			

ANALYSIS

The first step in the analysis was to compute the pooled variance and co-variance of these three variables for each of the categories by adding the sum of the squares and the sum of cross products in each category. This was then divided by the total number of degrees of freedom, namely the sum of the number of observations in the contrail category, plus the number of observations in the no-contrail category minus two. The mean of each of the variables must be eliminated so that the sum of squares and the sum of cross products are about a mean of zero.

The pooled variances and co-variances for the above data are shown in the first three columns of Table II. The fourth column, is the difference between the averages of variable -45, -50 and -55 for the two situations.

TABLE II

	-45°C	-50°C	-55°C	
-45°C	0.657675	0.114825	-0.015626	-1.558
-50°C	0.114325	1.034075	0.421924	-2.998
-55°C	-0.015626	-.421924	1.322178	-3.466

The values of Table II are actually the coefficients of three linear equations which are solved for operators L_1, L_2 and L_3. The variables are multiplied by these operators to maximize the ratio of the variability between the two categories, as compared to the variability within categories.

ANALYSIS (con't)

The equation for determining the operators are:

(1) $(0.657675)L_1 + (0.114825)L_2 + (-0.015626)L_3 = -1.558$

(2) $(0.114825)L_1 - (1.034975)L_2 - (0.421924)L_3 = -2.998$

(3) $(-0.015626L_1 - (0.421924)L_2 - (1.322178)L_3 =$ [illegible].466

therefore:

$L_1 = 2.099912, L_2 = 1.821953, L_3 = 2.064852$

substitute these values in equations (1), (2), and (3) above and compute the sum for each case.

The same operator is used on both the contrail and no-contrail data producing a set of numbers. When good discrimination is present, one category will produce numbers of an order of magnitude different from that of any other. In this study, such a point was computed which separated the "contrail" category from the "no-contrail" category.

The discriminator values obtained in this manner for Pittsburgh are shown in Table III. The separation point was determined to be twenty (20), on which basis 98% of the forecasts are correctly discriminated.

The discriminant values for each station studied had a definite point of separation. (See table IV) Considering all cases, the accuracy of the predictions of "Contrail" vs"No Contrail" ranged from 85% to 100%. This table, in addition to providing the separation point for each station and the degree of accuracy obtained, gives the temperature points used and the operators computed for each station.

In utilizing this technique for forecasting occurrence of contrails, three operators and a separation point for the discriminant values have been determined. If "universal values" for both operators and separation points could be derived the forecasting technique would be greatly simplified.

To allow comparison of these operators, the values were normalized so that the sum of the L_1, L_2, and L_3 equaled a minus one. The separation points were also normalized. The close similiarity between stations, indicated that an average set of both operators and discriminant value might very well be developed which would be valid for all locations. This averaging yielded the following values:

TABLE III

DISCRIMINANT VALUE		DISCRIMINANT VALUE	
35.4	Contrail	24.0	Contrail
33.4	"	22.9	"
32.7	"	22.3	No Contrail
31.8	"	22.1	Contrail
31.8	"	20.8	"
31.1	"	19.4	No Contrail
31.1	"	19.3	"
31.0	"	17.2	"
10.6	"	16.2	"
30.3	"	13.1	"
29.5	"	12.9	"
29.4	"	12.5	"
29.3	"	12.4	"
29.2	"	10.6	"
29.1	"	10.6	"
28.5	"	9.7	"
28.3	"	9.4	"
27.9	"	8.8	"
27.7	"	8.2	"
26.8	"	8.1	"
24.2	"	8.0	"

TABLE IV

Station	1000' above & below following temperature levels			Operators			Separation Point	% Accuracy
				L_1	L_2	L_3		
Albuquerque	-50	-55	-60	-10.60	-2.92	0.52	46.0	100
Andrews AFB	-45	-50	-55	0.16	-1.94	-2.72	12.0	100
Buffalo	-45	-50	-55	- 2.33	-0.16	-1.11	13.0	90
Caribou	-45	-50	-55	- 0.54	-0.64	0.02	3.8	85
Geiger	-45	-50	-55	- 0.82	-1.39	-2.71	12.5	95
Great Falls	-45	-50	-55	- 1.15	-1.18	-0.03	5.0	86
Nantucket	-45	-50	-55	- 0.86	-0.47	-0.41	4.7	85
Pittsburg	-45	-50	-55	- 2.10	-1.82	-2.06	14.0	98
Rapid City Ellsworth	-45	-50	-55	- 0.21	-0.65	-1.14	9.5	92
Davis-Monthan AFB	-50	-55	-60	- 0.85	-0.52	-0.81	6.0	88
Griffis AFB	-45	-50	-55	1.38	-1.02	-1.81	1.5	93
Lakehurst NAS	-45	-50	-55	0.07	-1.19	-5.78	15.0	100
Longbeach, Calif	-45	-50	-55	- 4.06	-0.04	-2.19	25.0	96
Mitchel AFB	-45	-50	-55	- 1.28	-1.34	-5.63	20.0	99
Oakland WBAS	-45	-50	-55	- 2.98	-1.13	-1.29	22.5	96
Seattle, Wash.	-45	-50	-55	1.48	-0.17	-4.47	0	98
Selfridge AFB	-45	-50	-55	0.59	-0.95	-2.79	7.0	98
Wright Patterson AFB	-45	-50	-55	0.03	-1.24	-0.54	6.4	90
Portland, Ore.	-45	-50	-55	0.50	0.61	-2.11	1.5	96

ANALYSIS (con't)

$L_1 = 0.14$
$L_2 = 0.32$
$L_3 = 0.54$
and a separation point of 2.8.

The effectiveness of these so called "universal values" in forecasting the occurrence or non=occurrence of contrails below 45 thousand feet was tested on the data for each station. It was found that the average values were equally effective at all stations. Therefore, by utilizing this mathematical technique which describes the slope of a temperature ascent curve, one can quite accurately forecast the formation of contrails.

FORECASTING TECHNIQUE

The proceedure to be used is as follows: from the latest raob,

(1) determine ΔT for 1000 feet above and below the levels at which the temperatures -45°C, -50°C and -55°C occur.

(2) From Table V, obtain the values corresponding to the ΔT for L_1, L_2 and L_3 (The purpose of this table was to eliminate unnecessary multiplication).

(3) Total these three values and if this sum is greater than 2.8, then contrails may be expected to develop, but if the sum is less than 2.8 then no contrails are expected.

TABLE V

ΔT L.R.	L_1 x -45°	L_2 x -50°	L_3 x -55°	ΔT L.R.	L_1 x -45°	L_2 x -50°	L_3 x -55°
-0.1	0.0	0.0	0.1	-3.6	0.5	1.2	1.9
-0.2	0.0	0.1	0.1	-3.7	0.5	1.2	2.0
-0.3	0.0	0.1	0.2	-3.8	0.5	1.2	2.1
-0.4	0.1	0.1	0.2	-3.9	0.5	1.2	2.1
-0.5	0.1	0.2	0.3	-4.0	0.6	1.3	2.2
-0.6	0.1	0.2	0.3	-4.1	0.6	1.3	2.2
-0.7	0.1	0.2	0.4	-4.2	0.6	1.3	2.3
-0.8	0.1	0.3	0.4	-4.3	0.6	1.4	2.3
-0.9	0.1	0.3	0.5	-4.4	0.6	1.4	2.4
-1.0	0.1	0.3	0.5	-4.5	0.6	1.4	2.4
-1.1	0.2	0.4	0.6	-4.6	0.6	1.5	2.5
-1.2	0.2	0.4	0.6	-4.7	0.6	1.5	2.5
-1.3	0.2	0.4	0.7	-4.8	0.7	1.5	2.6
-1.4	0.2	0.4	0.8	-4.9	0.7	1.6	2.6
-1.5	0.2	0.5	0.8	-5.0	0.7	1.6	2.7
-1.6	0.2	0.5	0.9	-5.1	0.7	1.6	2.8
-1.7	0.2	0.5	0.9	-5.2	0.7	1.7	2.8
-1.8	0.3	0.6	1.0	-5.3	0.7	1.7	2.9
-1.9	0.3	0.6	1.0	-5.4	0.8	1.7	2.9
-2.0	0.3	0.6	1.1	-5.5	0.8	1.8	3.0
-2.1	0.3	0.7	1.1	-5.6	0.8	1.8	3.0
-2.2	0.3	0.7	1.2	-5.7	0.8	1.8	3.1
-2.3	0.3	0.7	1.2	-5.8	0.8	1.8	3.1
-2.4	0.3	0.8	1.3	-5.9	0.8	1.9	3.2
-2.5	0.4	0.8	1.4	-6.0	0.8	1.9	3.2
-2.6	0.4	0.8	1.4	-6.1	0.9	2.0	3.3
-2.7	0.4	0.9	1.5	-6.2	0.9	2.0	3.3
-2.8	0.4	0.9	1.5	-6.3	0.9	2.0	3.4
-2.9	0.4	0.9	1.6	-6.4	0.9	2.1	3.4
-3.0	0.4	1.0	1.6	-6.5	0.9	2.1	3.5
-3.1	0.4	1.0	1.7	-6.6	0.9	2.1	3.6
-3.2	0.4	1.0	1.7	-6.7	0.9	2.1	3.6
-3.3	0.5	1.1	1.8	-6.8	1.0	2.2	3.7
-3.4	0.5	1.1	1.8	-6.9	1.0	2.2	3.7
-3.5	0.5	1.1	1.9	-7.0	1.0	2.2	3.8

TABLE V
(con't)

ΔT L.R.	L_1 x -45°	L_2 x -50°	L_3 x -55°
-7.1	1.0	2.3	3.8
-7.2	1.0	2.3	3.9
-7.3	1.0	2.3	3.9
-7.4	1.0	2.4	4.0
-7.5	1.1	2.4	4.1
-7.6	1.1	2.4	4.1
-7.7	1.1	2.5	4.2
-7.8	1.1	2.5	4.2
-7.9	1.1	2.5	4.3
-8.0	1.1	2.6	4.3
-8.1	1.1	2.6	4.4
-8.2	1.1	2.6	4.4
-8.3	1.2	2.7	4.5
-8.4	1.2	2.7	4.5
-8.5	1.2	2.7	4.6
-8.6	1.2	2.8	4.6
-8.7	1.2	2.8	4.7
-8.8	1.2	2.8	4.8
-8.9	1.2	2.8	4.8
-9.0	1.3	2.9	4.9
-9.1	1.3	2.9	4.9
-9.2	1.3	2.9	5.0
-9.3	1.3	3.0	5.0
-9.4	1.3	3.0	5.1
-9.5	1.3	3.0	5.1
-9.6	1.3	3.1	5.2
-9.7	1.4	3.1	5.2
-9.8	1.4	3.1	5.3
-9.9	1.4	3.2	5.3
-10.0	1.4	3.2	5.4

FORECASTING ALTITUDE OF CONTRAIL FORMATION

With a method established of forecasting the existence or non existence of contrails, the next stop was to forecast the altitude at which the contrails would occur. Indications were that the base of the contrail might be directly related to such meteorological variables (upper air parameters) as tropopause height and temperature, humidity of surrounding air, turbulence and location of jet stream core, lapse rates at various levels and perhaps the relative location of the general circulation patterns at constant pressure levels, say above 300 mbs.

As mentioned previously, "Project Cloud Trail" data was used. Altitudes of all phenomena were reported in thousands of feet according to the ship's altimeter. The reported altitudes were converted to pressure altitude.

RAOB's were now plotted for all the data cards for which altimeter settings were available. Following discussions on properties affecting the commencement altitude of the contrail formation, it was felt that the one atmospheric phenomena that most affected contrails was the tropopause. In an attempt to determine how the tropopause affected formation, graphs of tropopause height versus contrail height, tropopause height versus contrail temperature, tropopause temperature versus contrail height and tropopause temperature versus contrail temperature were plotted for a number of stations. Figures I thru IV are the plots for Tuscon, Arizona which are representative of all stations in this study.

FigureI. The Temperature of the contrail base varies within a small area regardless of season; while the tropopause temperature decreases with season, being colder in the summer and warmer in the winter.

Figure II. The tropopause height varies with the season while the contrail temperature remains with in a small range.

Figure III. Illustrates the relationship between contrail height and tropopause temperature. Although a relationship exists, it would necessitate taking temperatures from a sounding with an unattainable degree of accuracy. Therefore, it was felt that using tropopause temperature in order to estimate contrail height was not the answer.

Figure I

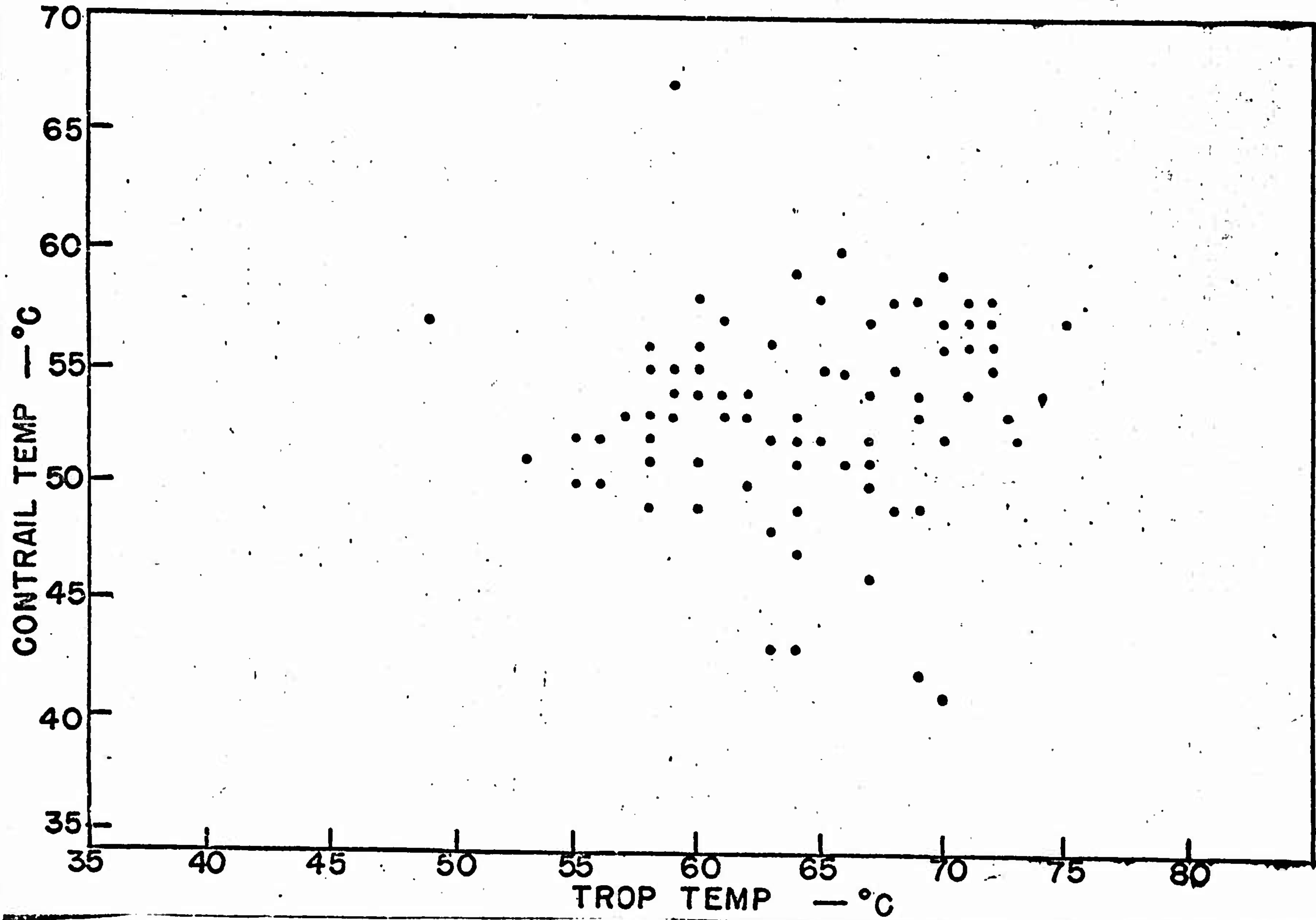
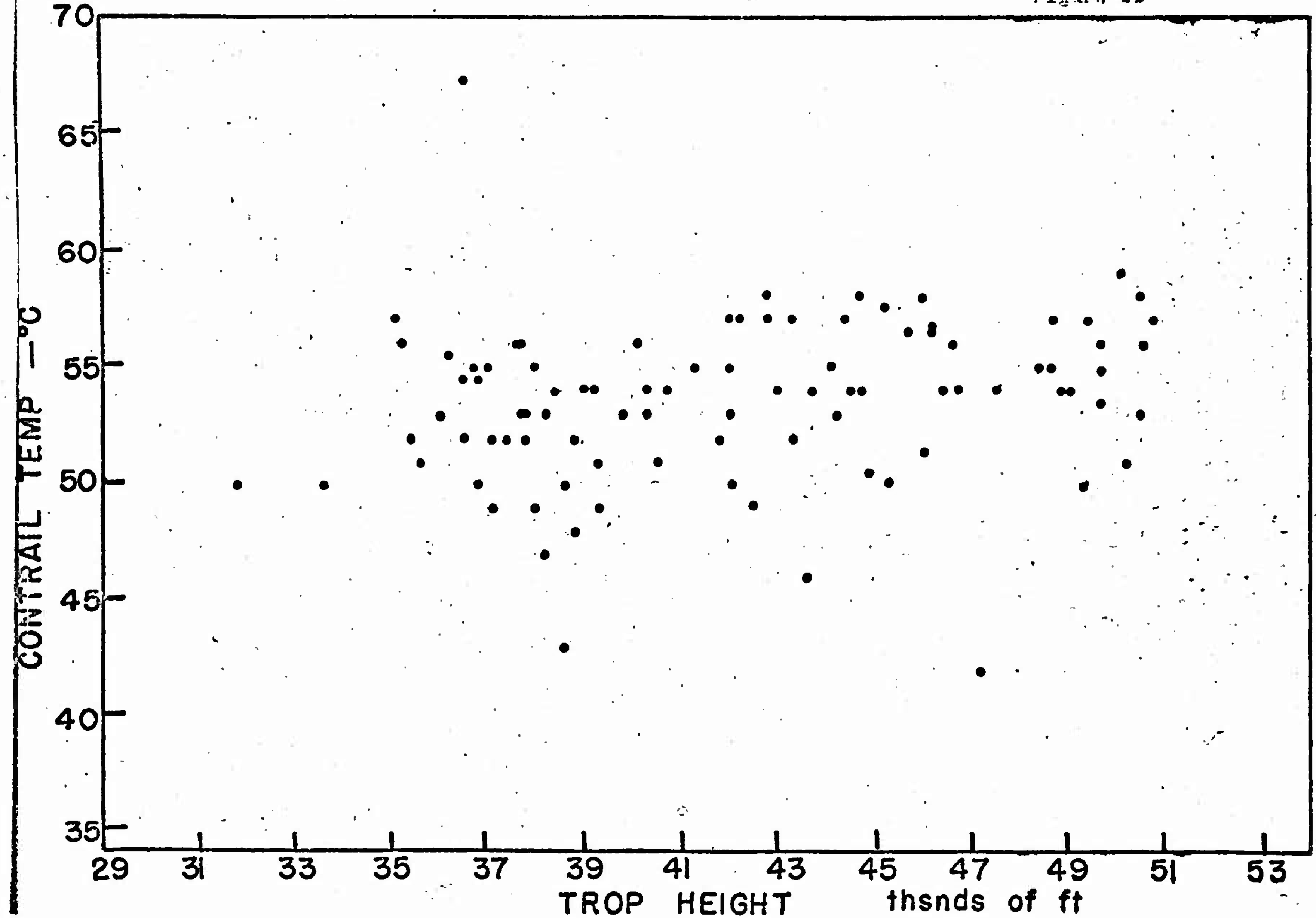
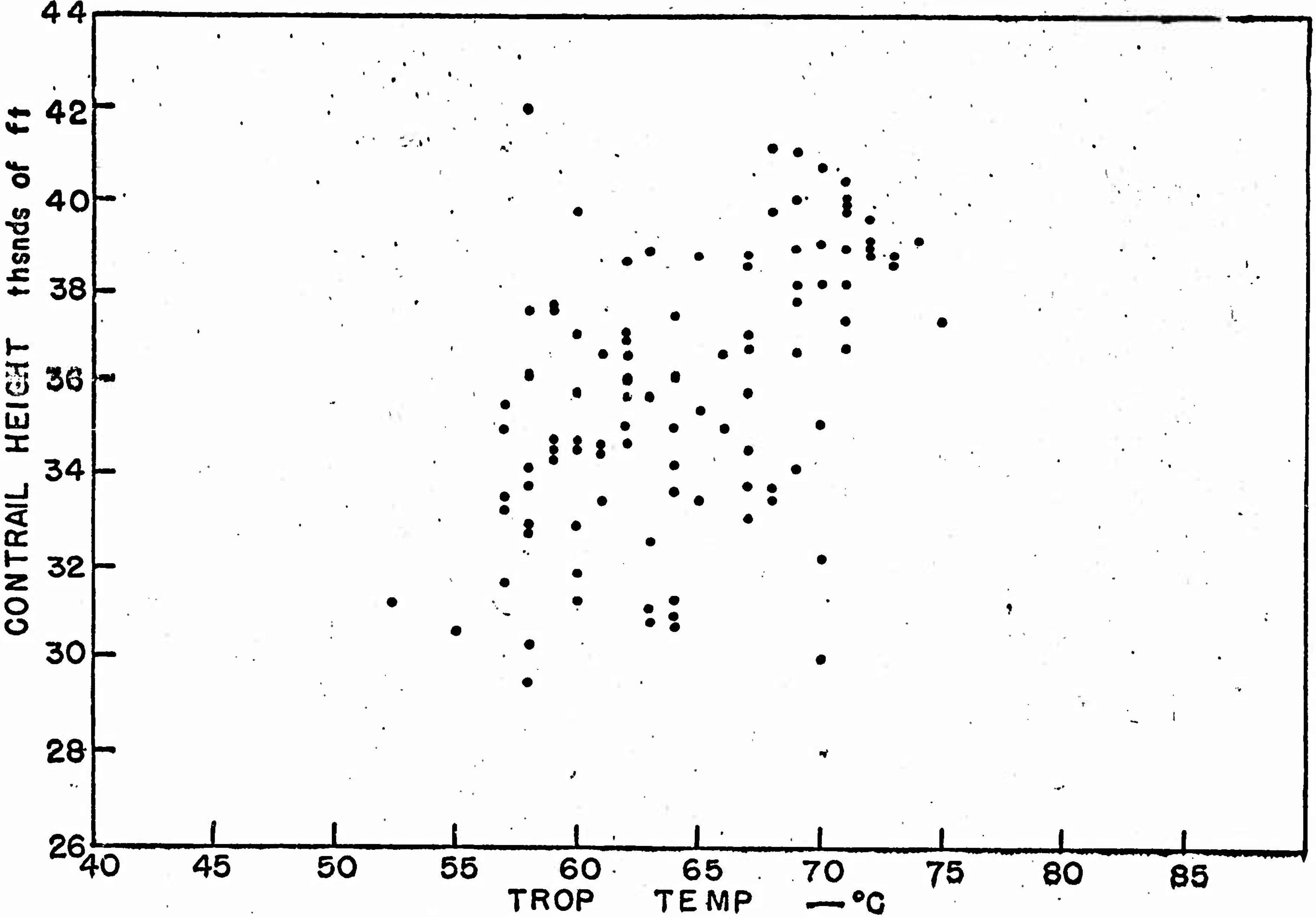


Figure I

Figure II



CONTRAIL TEMP —°C
70
65
60
55
50
45
40
35
29
31
33
35
37
39
41
43
45
47
49
51
53
TROP HEIGHT thsnds of ft


CONTRAIL HEIGHT thsnds of ft
44
42
40
38
36
34
32
30
28
26
40
45
50
55
60
65
70
75
80
85
TROP TEMP —°C

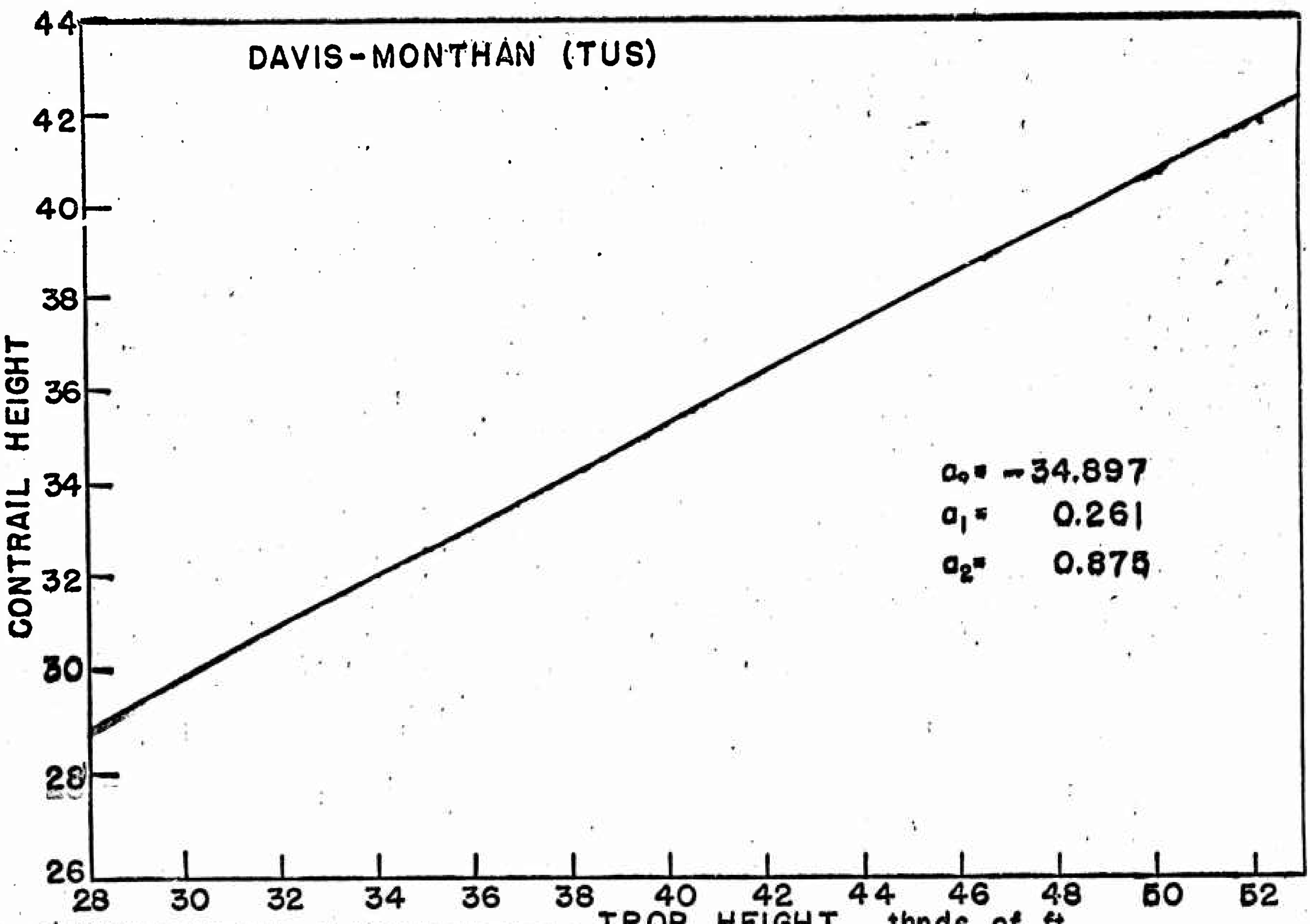

DAVIS-MONTHAN (TUS)
a_0 = −34.897
a_1 = 0.261
a_2 = 0.878
CONTRAIL HEIGHT
TROP HEIGHT thnds of ft
44
42
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
44
46
48
50
52

Figure IV. The relationship between the tropopause height and the altitude at which contrails formed approximated a straight line. It's slope was dependent on the season of the year. Figure IV became the basis of the first parameter. Similiar graphs were plotted for all remaining Air Force Bases with the exception of Limestone AFB and Kinross AFB for which insufficient data was available. (Copies of all graphs are at end of report).

The first step in estimating contrail height was to locate the prevailing height of the tropopause on Figure IV, which in turn will indicate the approximate height of the contrail formation. This proved quite sucessful. However, no one parameter is sufficient in itself to forecast heights of contrail formation therefore a second parameter was needed which, when taken jointly with the first would serve as a satisfactory basis for forecasting the altitude of contrail formation.

An attempt was made to relate the vertical location of the jet stream core to the base altitude of contrails. It was found that vertical movement of the jet stream did affect the contrail height, however the tropopause splits at the jet core, with the tropopause height to the left of the jet axis being lower than on the right. In order not to re-introduce this effect into any computations the jet stream was eliminated from the study.

No measured humidity data was available. However the presence or lack of moisture has been accounted for in the forecast, by the behavior of the lapse rate. A stable lapse rate inferring descending and therefore dry air while an unstable lapse rate implies ascending and therefore moist air.

300mb, 200mb, and 150mb, facimile maps were used to study the effects of the large scale circulation patterns on formation. This was accomplished by plotting the relative distance of the center of troughs in degrees longitude east or west of the station in question. It was felt that such large scale motions could not be related to the height fluctuations of contrail formation.

A variable which would reflect lapse rate over a period of time was worth investigating. Contrails, it would appear, occur in most cases at high altitudes and therefore at very cold temperatures.

Previous studies have established that with a temperature below -50°C the relative frequency of visible contrails increases markedly. For our purpose the height at which -50°C first occurs was taken as a separation point for visible and non visible contrails. Of course the bases of contrail formation do not always occur at heights above -50°C, but instances of bases occurring at warmer temperatures are generally explained by sudden drops in the tropopause height. Thus -50°C represents a limit below which strong contrails will generally occur. The height at -50°C when viewed over a few successive soundings does reflect a change in lapse rate.

Our theory with regard to the parameters affecting contrail formation was that relatively few parameters significantly effected the formation while a myriad of parameters affect formation to a minor degree. With this in mind, further investigation of additional parameters was halted. The two basic parameters were tested by running a linear correlation.

(1) $Z' = a_0 + a_1 X_1 + a_2 X_2$

Where:

Z' is the predicted contrail height,
X_1 is the estimated contrail height determined from Figure IV.
X_2 is the height at which -50°C occurs on the latest sounding.

The a_0, a_1 and a_2 are constants derived from the distribution of the contrail data for each air base.

In the case of Pittsburg, 52 cases of both strong and moderate contrails were used to determine the distribution of Z, the contrail height obtained from "Project Cloud Trail". The equations of least square are:

$$a_0 n + a_1 \sum X_1 + a_2 \sum X_2 = \sum Z$$

$$a_0 \sum X_1 + a_1 \sum X_1^2 + a_2 \sum X_1 X_2 = \sum Z X_1$$

$$a_0 \sum X_2 + a_1 \sum X_1 X_2 + a_2 \sum X_2^2 = \sum Z X_2$$

Their solution when substituted into (1) yields the desired polynomial of best fit.

Table VI illustrates the percent reduction acheived for each station when this method is tested using the observed data from the Cloud Trail cards and the RAOBS.

TABLE VI

AIR FORCE BASE-UPPER AIR STATION	PER CENT REDUCTION	SAMPLES SIZE
KIRKLAND AFB (ABQ)	82.0	118
GEIGER AFB (GEG)	94.3	58
DAVIS-MONTHAN AFB (TUS)	75.3	41
OTIS AFB (ACK)	79.4	40
GREAT FALLS AFB (GTF)	78.1	35
LANGELEY AFB (NGU)	73.1	25
ELLSWORTH AFB (RAP)	59.8	94
McCHORD AFB (SEN)	95.0	47
McGUIRE AFB (HEM)	95.8	50
MITCHELL AFB (HEM)	94.5	81
PORTLAND NATIONAL AFB (PDX)	91.7	38
NIAGARA AFB (BUF)	93.6	62
HAMILTON AFB (OAK)	80.5	44
ANDREWS AFB (DCA)	85.3	74
OXNARD AFB (LGB)	86.1	88
WRIGHT-PATTERSON AFB (FFO)	84.4	54
PITTSBURG NATIONAL AFB (PIT)	96.4	52
GRIFFITHS AFB (RME)	93.6	44
SELFRIDGE AFB (MTC)	85.8	64

Having developed a "universal equation" for forecasting the occurrence of contrails, it was thought advisible to attempt a similiar "universal equation" for forecasting the altitude at which contrails would develop.

a_0, a_1, and a_2 were averaged for all stations and the resulting values, $a_0 = -60.4$, $a_1 = +0.41$, and $a_2 = +0.80$ were tested to determine the effect on the previously calculated percent reductions based on constants derived for each particular station. Table VII gives the comparison of the original percent reductions to the percent reduction obtained from using averaged values. It can be seen that the new method reduced the percent reduction to a minor degree, namely about 2% in most cases. Therefore, an averaged equation can be used to forecast the height of contrail formation regardless of the location of the station.

The forecasting proceedure developed thus far is:

1. From the current sounding determine the altitude at which -50°C occurs. This value is X_2.

2. From same sounding obtain tropopause altitude.

3. From figure IV obtain the estimated contrail height which is then X_1.

4. The altitude of contrail formation can then be forecasted using;

$$(1)\quad Z' = a_0 + a_1 X_1 + a_2 X_2$$

in which:

$a_0 = -60.4$
$a_1 = 0.41$
$a_2 = 0.80$

Since the relationship between the measured tropopause height (H) for a particular day and the estimated contrail height (X_1) for that day is linear, then

$$X_1 = e_0 + e_1 H$$

Therefore substituting this in equation (1) above for each station

$$Z' = a_0 + a_1 e_0 + a_1 e_1 H + a_2 X_2$$

results. Letting

$A = (a_0 + a_1 e_0)$ $\qquad B = a_1 e_1$ $\qquad C = a_2$

TABLE VII

Station:	% Reduction Obtained from Values for Individual Stations:	% Reduction from Averaged Values for all Stations:
PIT:	96.4	95.9
GEG:	94.3	93.3
NGU:	73.1	71.3
RME:	93.6	92.9
MTC:	85.8	84.5
OAK:	80.5	77.7
GTF:	78.1	76.4
PDX:	91.7	90.8
SEN:	95.0	93.9
HEM:	94.5	93.6
ADW:	85.3	83.3
WRI:	96.0	96.0
BUF:	93.6	95.0
TUS:	75.3	73.2
LGB:	86.1	81.4

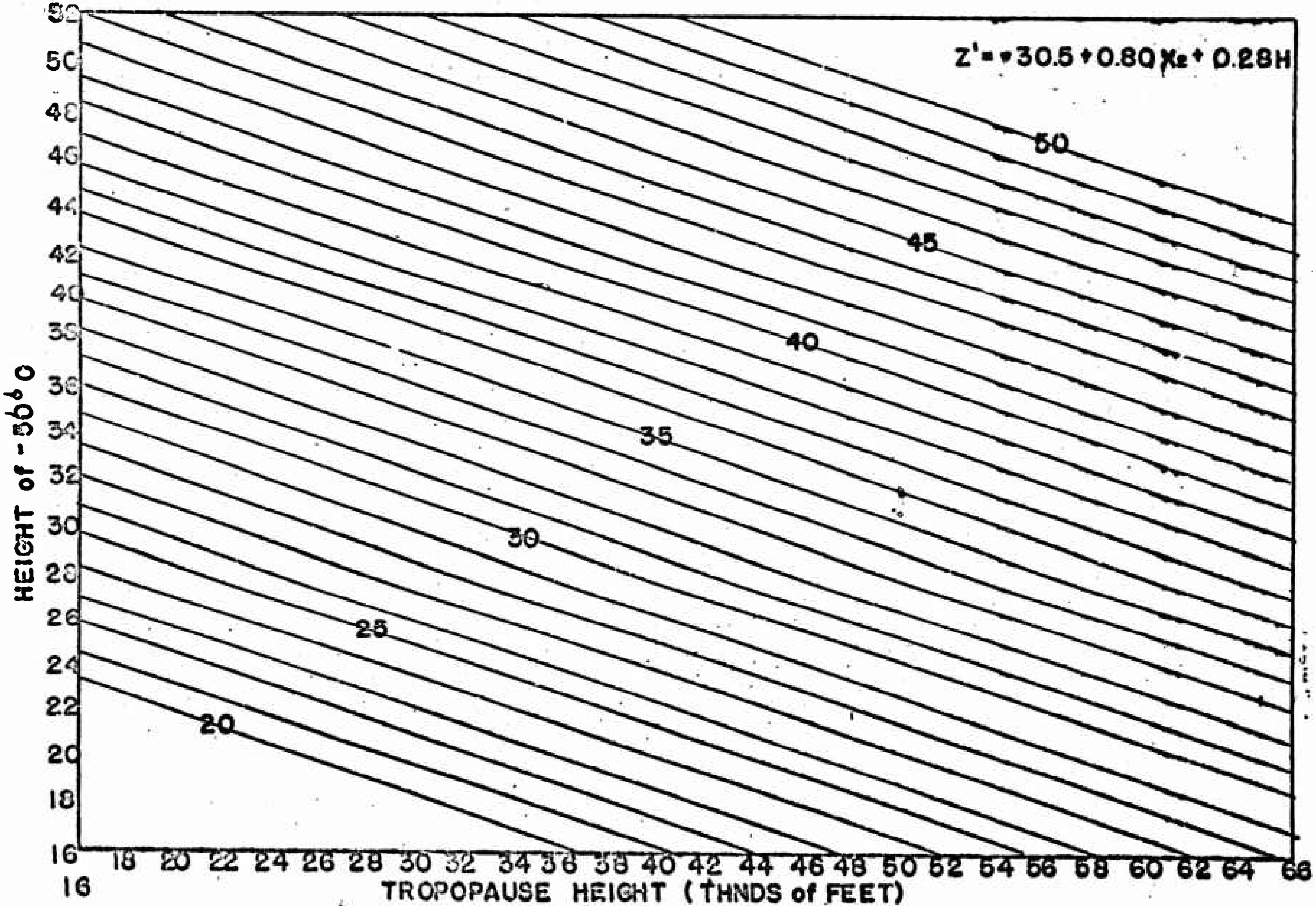
$a_0 = -60.4$
$a_1 = +0.41$
$a_2 = +0.80$

Then: $Z' = A + BH + CX_2$

Averaging these computations for each station resulted in a equation requiring only tropopause height and the height at which -50°C occurs thus eliminating the necessity of step #3 above, that is, estimating contrail height. This average equation is:

$$Z' = -30.5 + 0.28H + 0.80X_2$$

Figure V represents a graph of this equation with the tropopause height (H) given as the abscissa and the height of the -50°C (X_2) given as the ordinate. The intersection of H and X_2 is Z' the height in thousands of feet at which contrails are likely to form.


Z' = -30.5 + 0.80 X₂ + 0.28H
50
45
40
35
30
25
20
HEIGHT of -30° C
16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66
TROPOPAUSE HEIGHT (THNDS of FEET)

SUMMARY AND EXAMPLE OF USAGE

The primary object of this study was to develop a mathematical method for forecasting both the development of contrails and the height at which they would form.

In forecasting the formation of contrails, three variables as slope measurement were used to characterize the general behavior of the ascent curve for a particular location. Utilizing these values discriminant analysis was carried out in which discriminants values for each station were determined. Each station studied had a definite point of separation between contrails and no contrails discriminants. The resulting operators and separation points were normalized and then averaged giving universal values for the operators and separation points which were effective, regardless of location.

In forecasting the altitude at which contrails would occur, two parameters were used, the height of the -50°C temperature and an estimated contrail height based on the pre-established linear relationship between it and the measured height of the tropopause. The equation developed was:

$$Z' = A + BH + CX_2$$

in which,

Z' = predicted height of the contrail
H = Height of the tropopause
X_2 = height of -50°C

An average equation for all stations studied was determined to be:

$$Z' = -30.5 + 0.28H + 0.80X_2$$

This equation was graphed and the intersection of H and X_2 gave the altitude of the contrail formation.

Using these two "universal equations" the following would be the proceedure used in forecasting:

$$L_1(\Delta T @ -45°) + L_2(\Delta T @ -50°) + L_3(\Delta T @ -55°) = D$$

where D = separator point

$$0.14(\Delta T @ -45°) + 0.32(\Delta T @ -50°) + 0.54(\Delta T @ -55°) = D$$

If D is $<$ 2.8 no contrails
D is $>$ 2.8, contrails will form

Therefore from Table V

EXAMPLE: ΔT at -45°C = -5
ΔT at -50°C = -3
ΔT at 55°C = -6

therefore the sum of the ΔT is 5.1 which is greater than D so it may be concluded that contrails will form.

Assuming from the latest sounding, that H the tropopause height was 46,000 feet and the X_2 the height of the -50°C was 40,000 feet, by using figure V we could conclude that the contrails forecasted to form should do so at approximately 41,800 feet.

RECOMMENDATIONS

A simple but valuable forecasting procedure has been developed as a result of this study.

It is therefore suggested that:

1. this method be tested throughout the country on independent data.

2. pertinent data be collected for altitudes above 45,000 feet to test the validity of this method.

3. if, data is already available then,

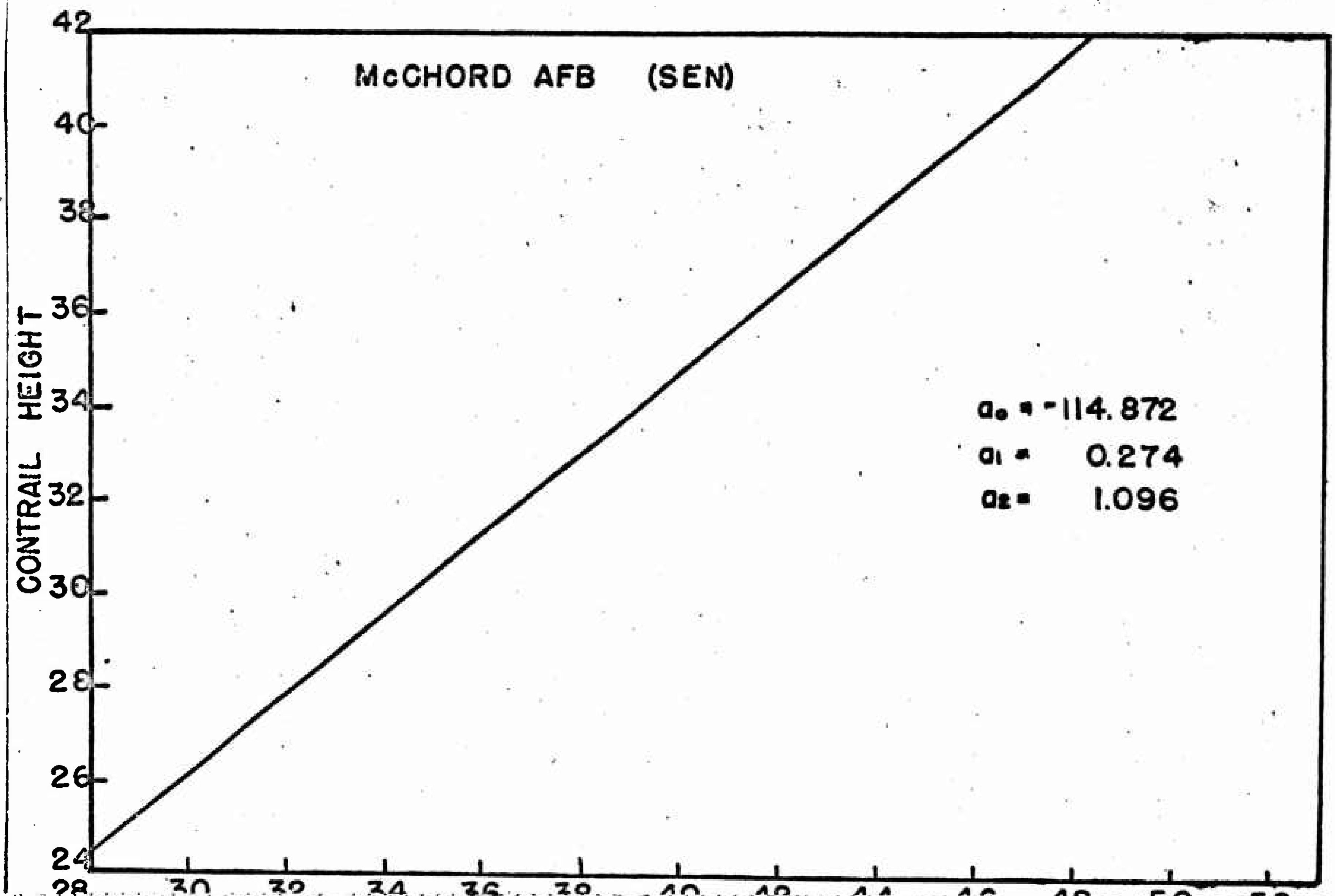
   a. this technique be tested on the new data

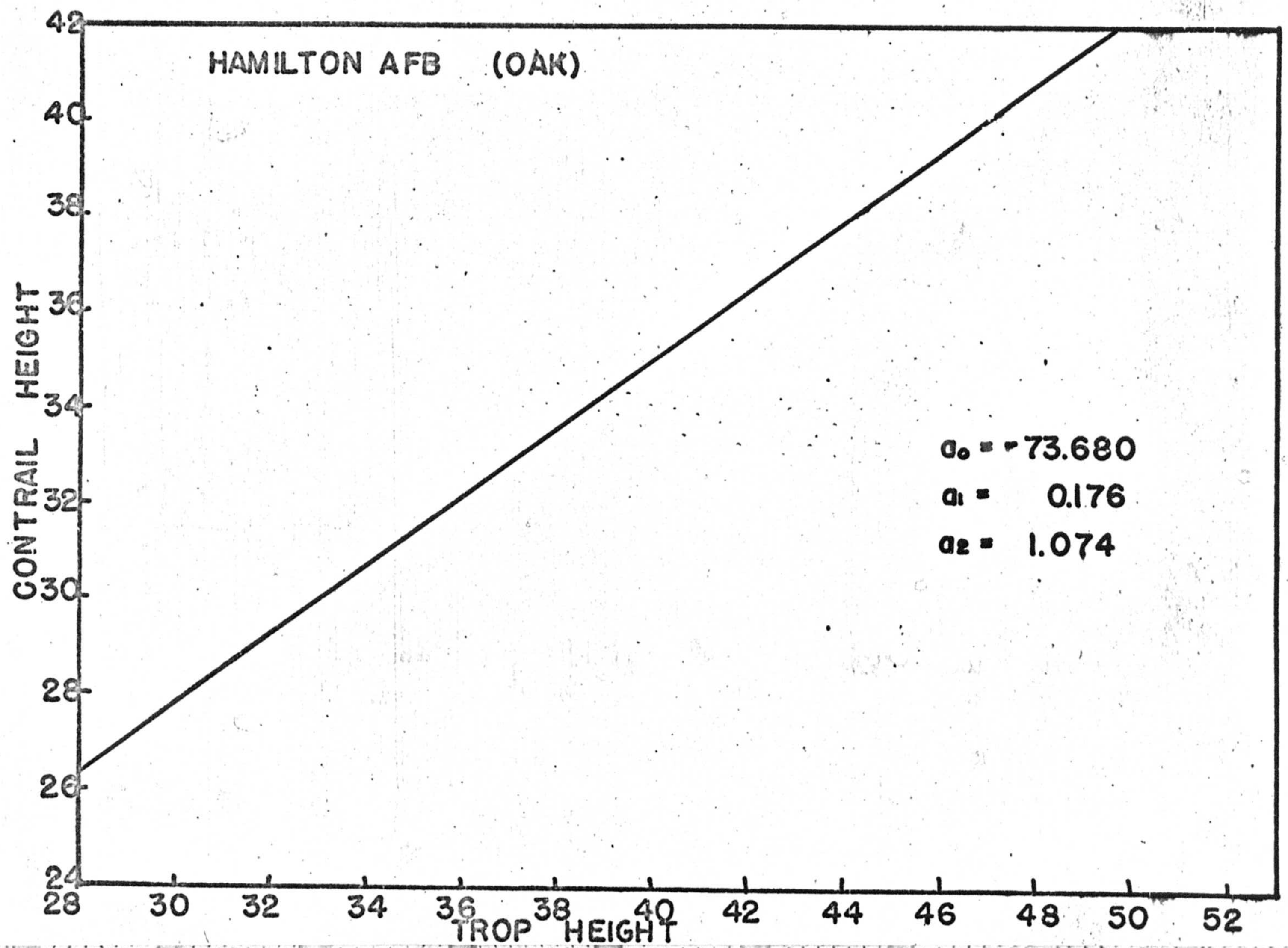
   b. changes be made for its improvement where necessary

   c. efforts be made to forecast the tops or endings of the contrails. In this study the endings occurred above the 45,000 foot altitude.

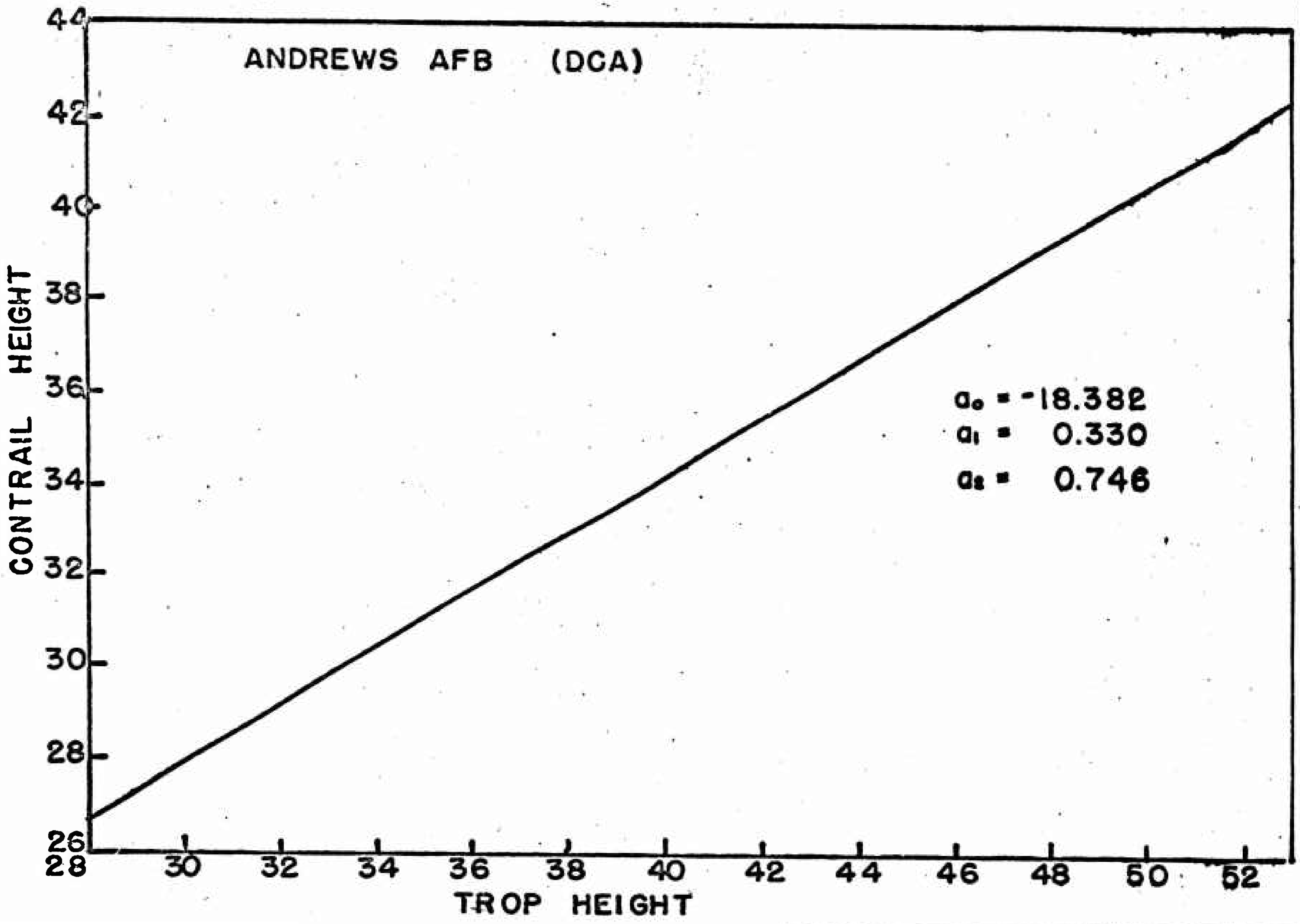
CONCLUSIONS

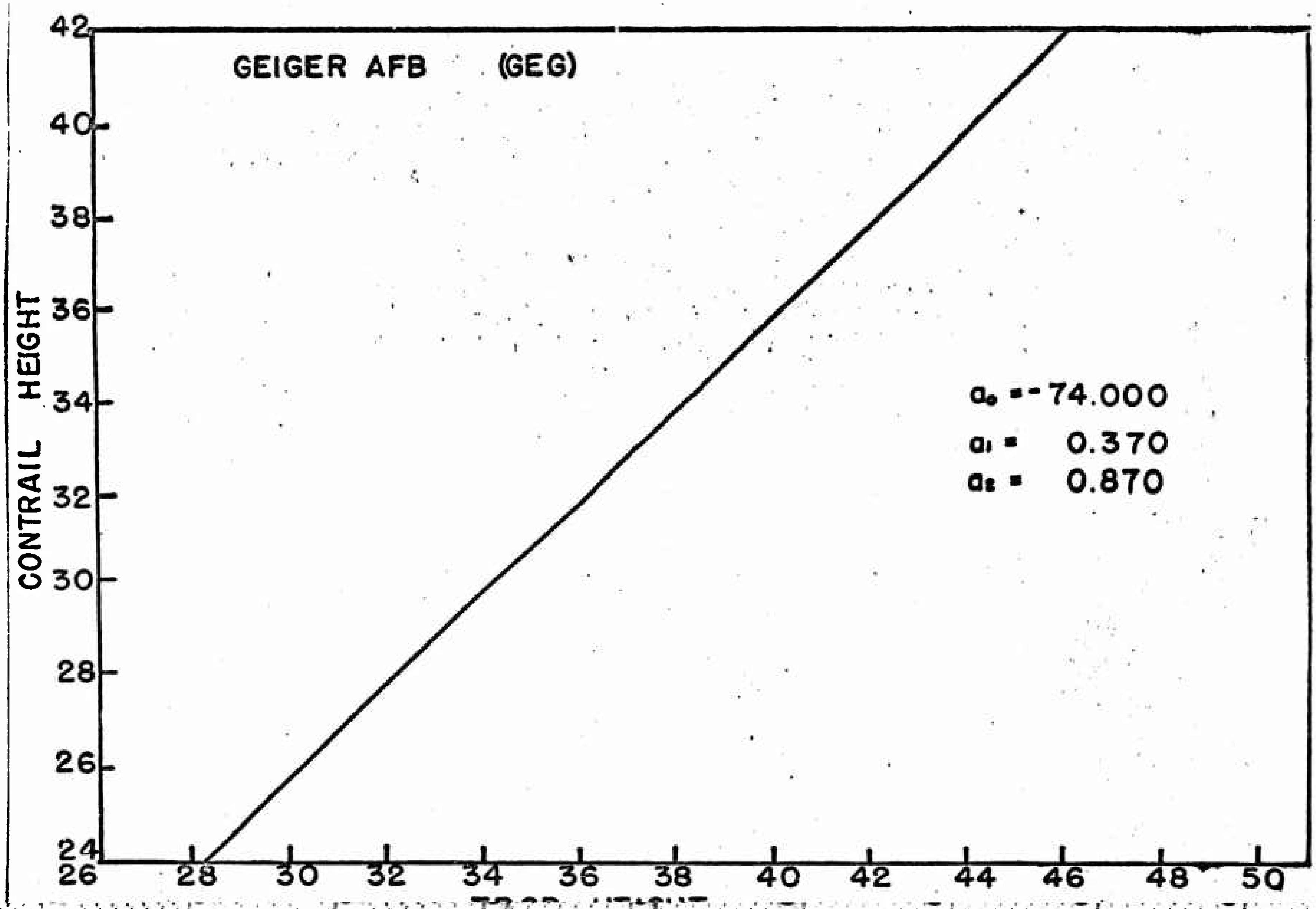
If the forecasting of contrails and the commencement altitudes are still of paramount importance, then the forecasting techniques developed as a result of this study should be valuable tools to the USAF.


McCHORD AFB (SEN)
CONTRAIL HEIGHT
42
40
38
36
34
32
30
28
26
24
a_0 = -114.872
a_1 = 0.274
a_2 = 1.096

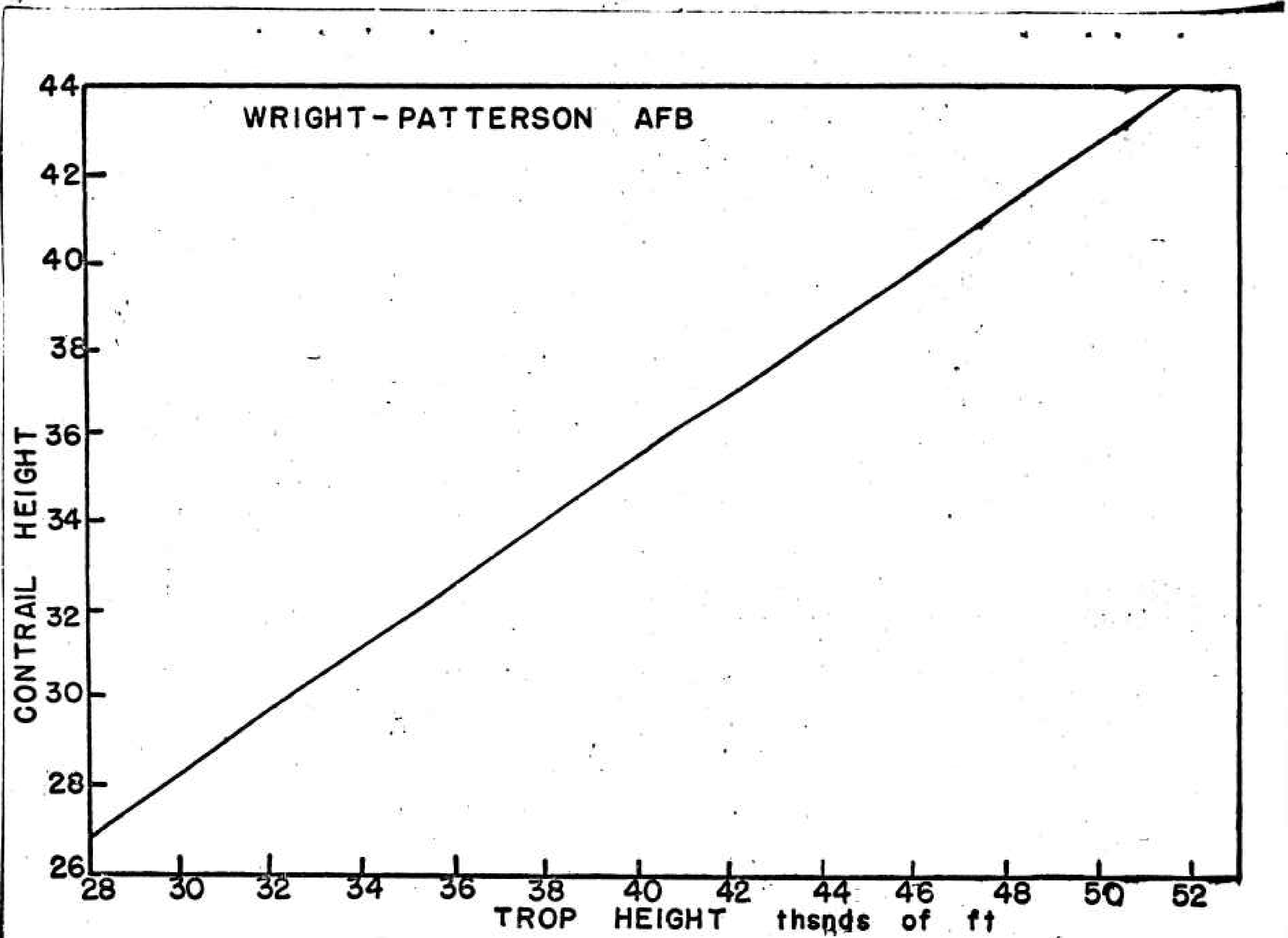


HAMILTON AFB (OAK)
a₀ = -73.680
a₁ = 0.176
a₂ = 1.074
CONTRAIL HEIGHT
TROP HEIGHT
24
26
28
30
32
34
36
38
40
42
28
30
32
34
36
38
40
42
44
46
48
50
52

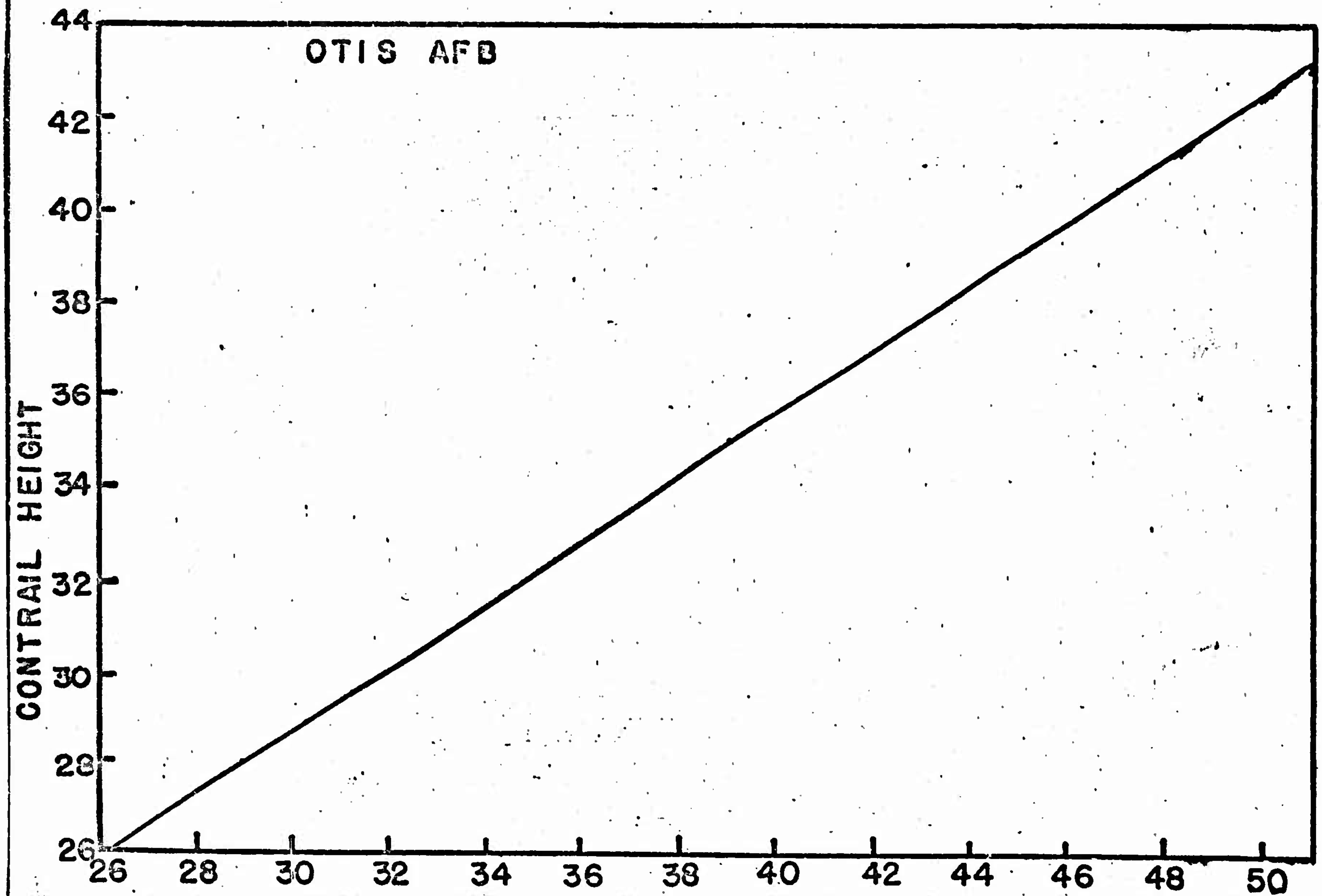

ANDREWS AFB (DCA)
a_0 = -18.382
a_1 = 0.330
a_2 = 0.746
CONTRAIL HEIGHT
26
28
30
32
34
36
38
40
42
44
TROP HEIGHT
28
30
32
34
36
38
40
42
44
46
48
50
52

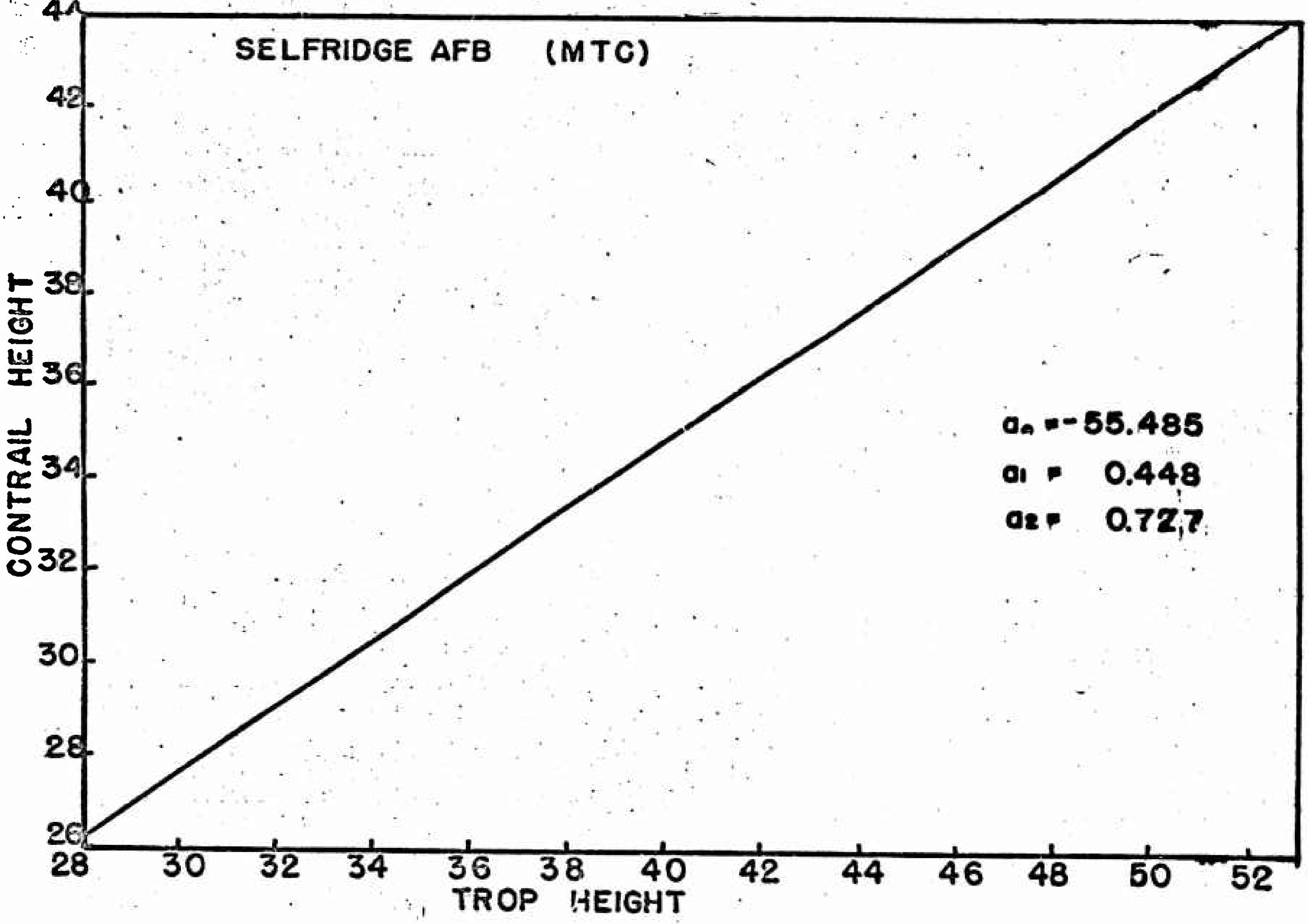


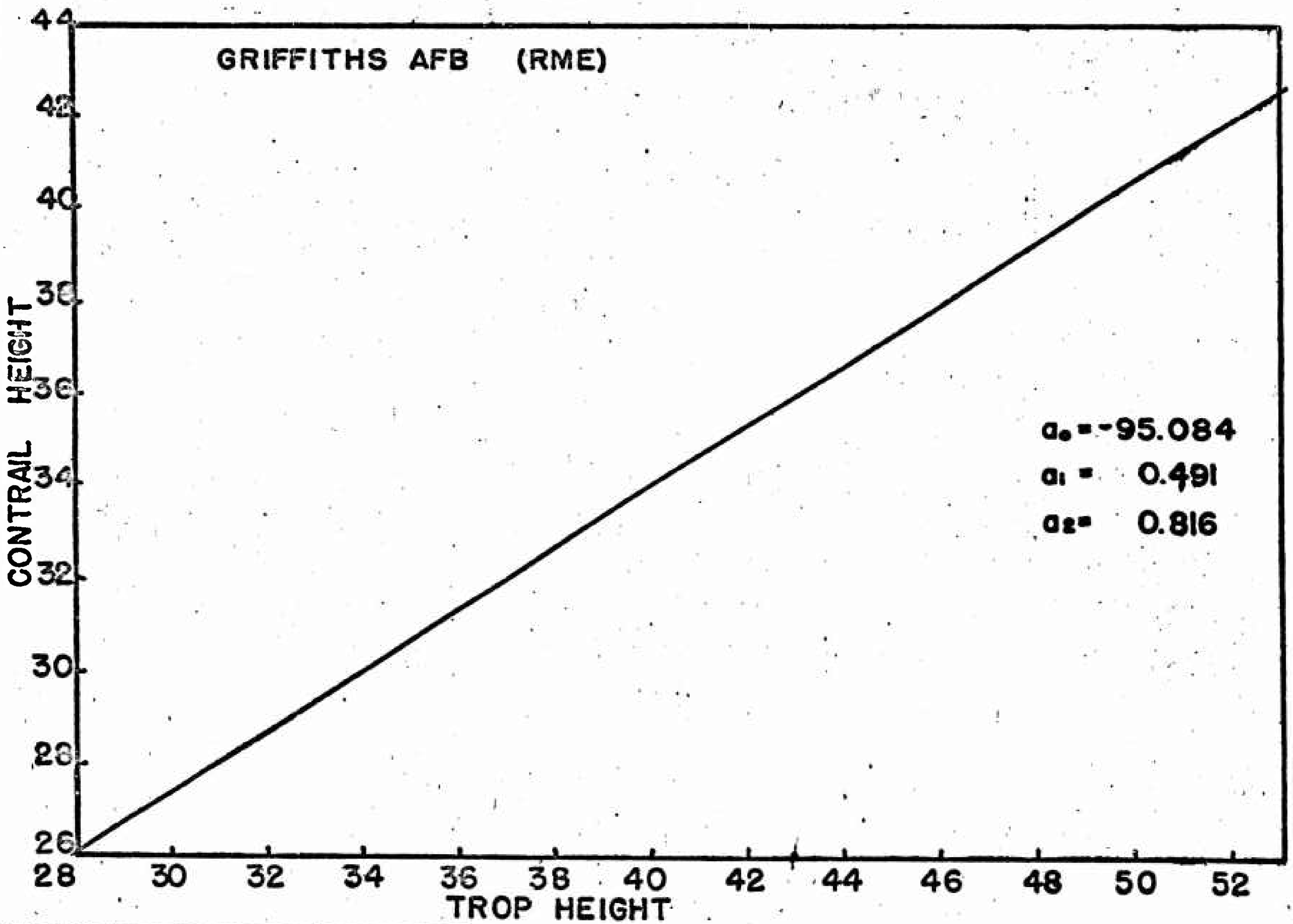
GEIGER AFB (GEG)
CONTRAIL HEIGHT
a_0 = -74.000
a_1 = 0.370
a_2 = 0.870
24
26
28
30
32
34
36
38
40
42
44
46
48
50



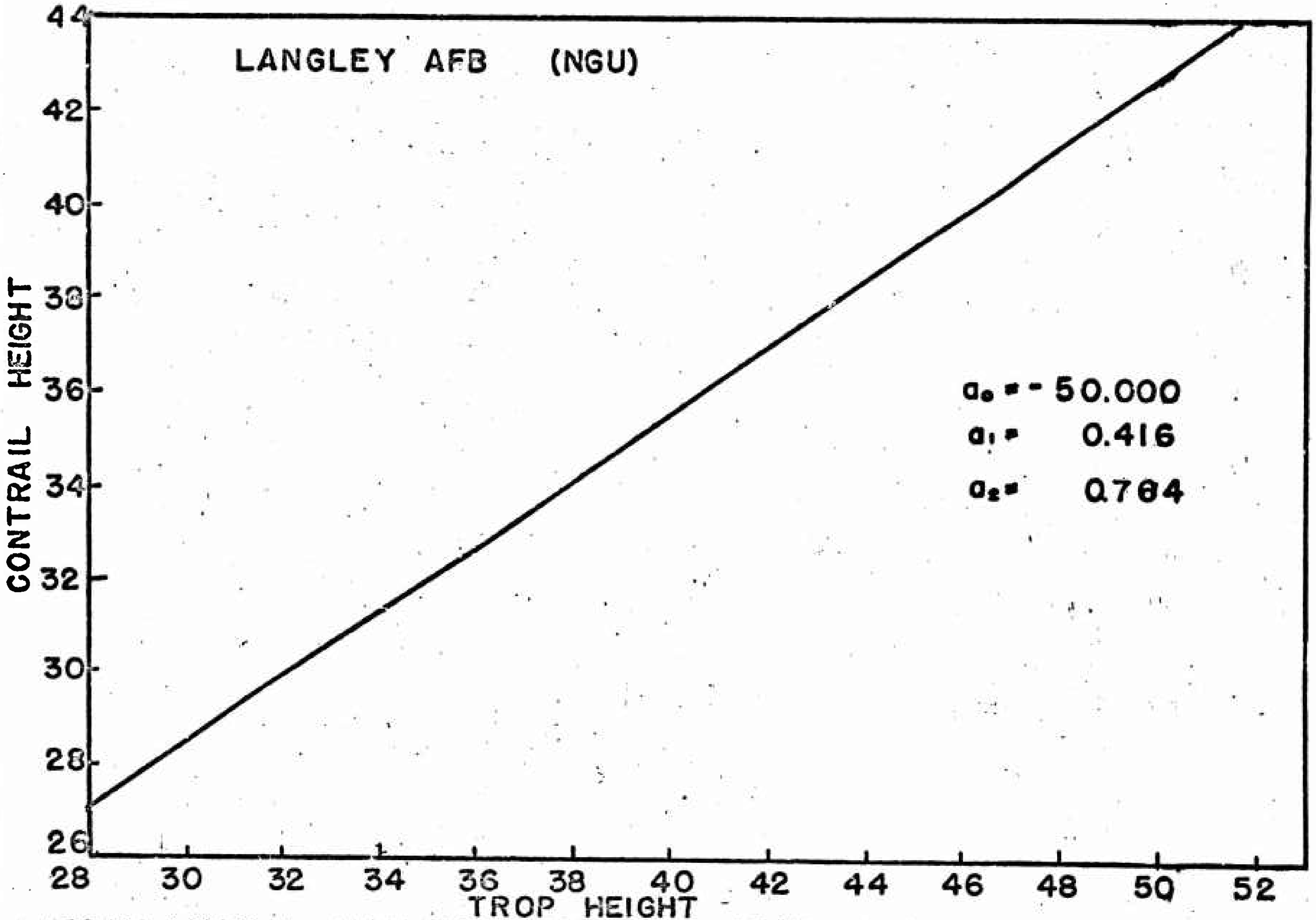
WRIGHT-PATTERSON AFB
CONTRAIL HEIGHT
44
42
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
44
46
48
50
52
TROP HEIGHT thsnds of ft

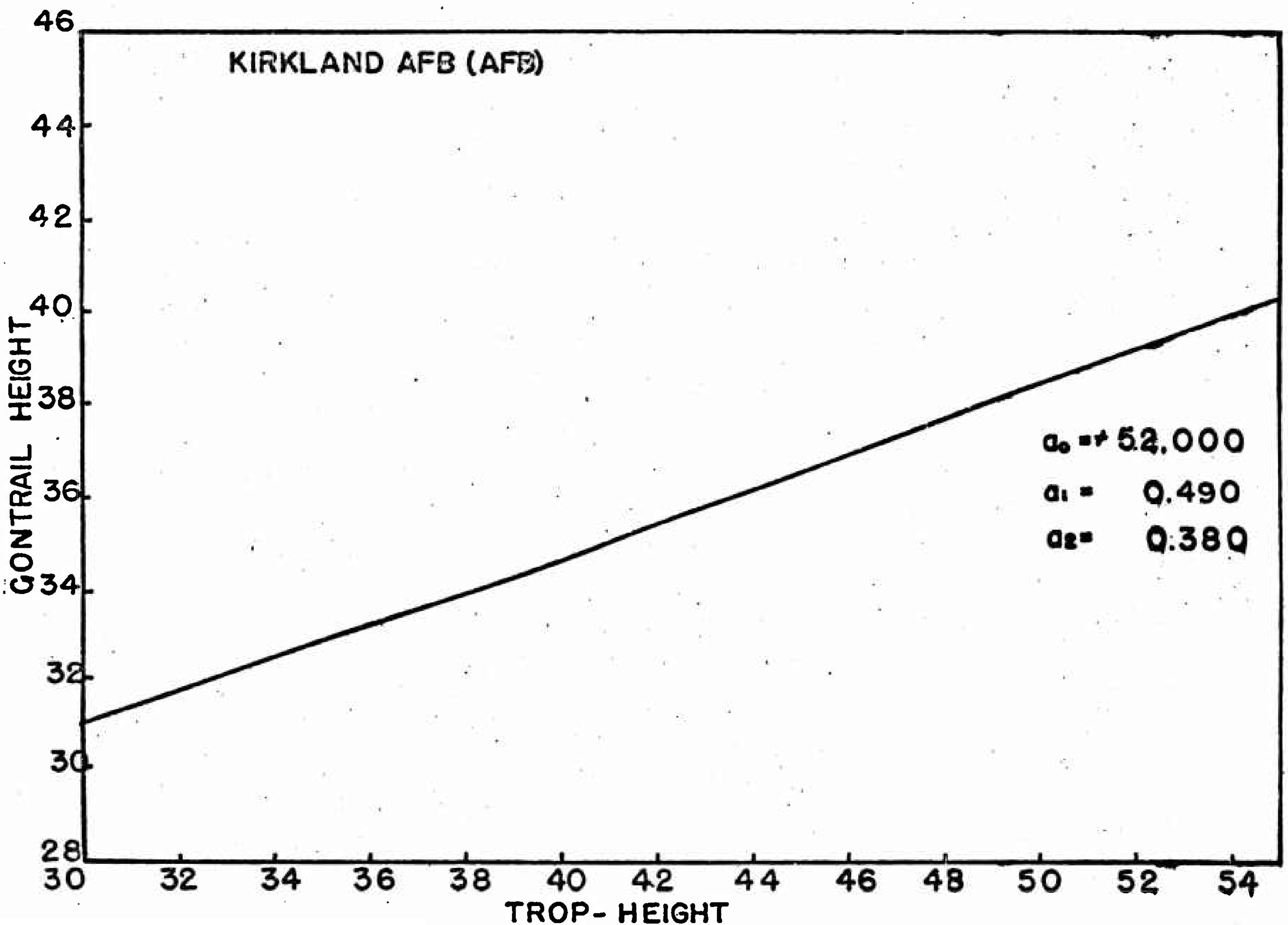

OTIS AFB
CONTRAIL HEIGHT
44
42
40
38
36
34
32
30
28
26
26
28
30
32
34
36
38
40
42
44
46
48
50


SELFRIDGE AFB (MTC)
a₀ = -55.485
a1 = 0.448
a2 = 0.727
CONTRAIL HEIGHT
TROP HEIGHT
44
42
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
44
46
48
50
52

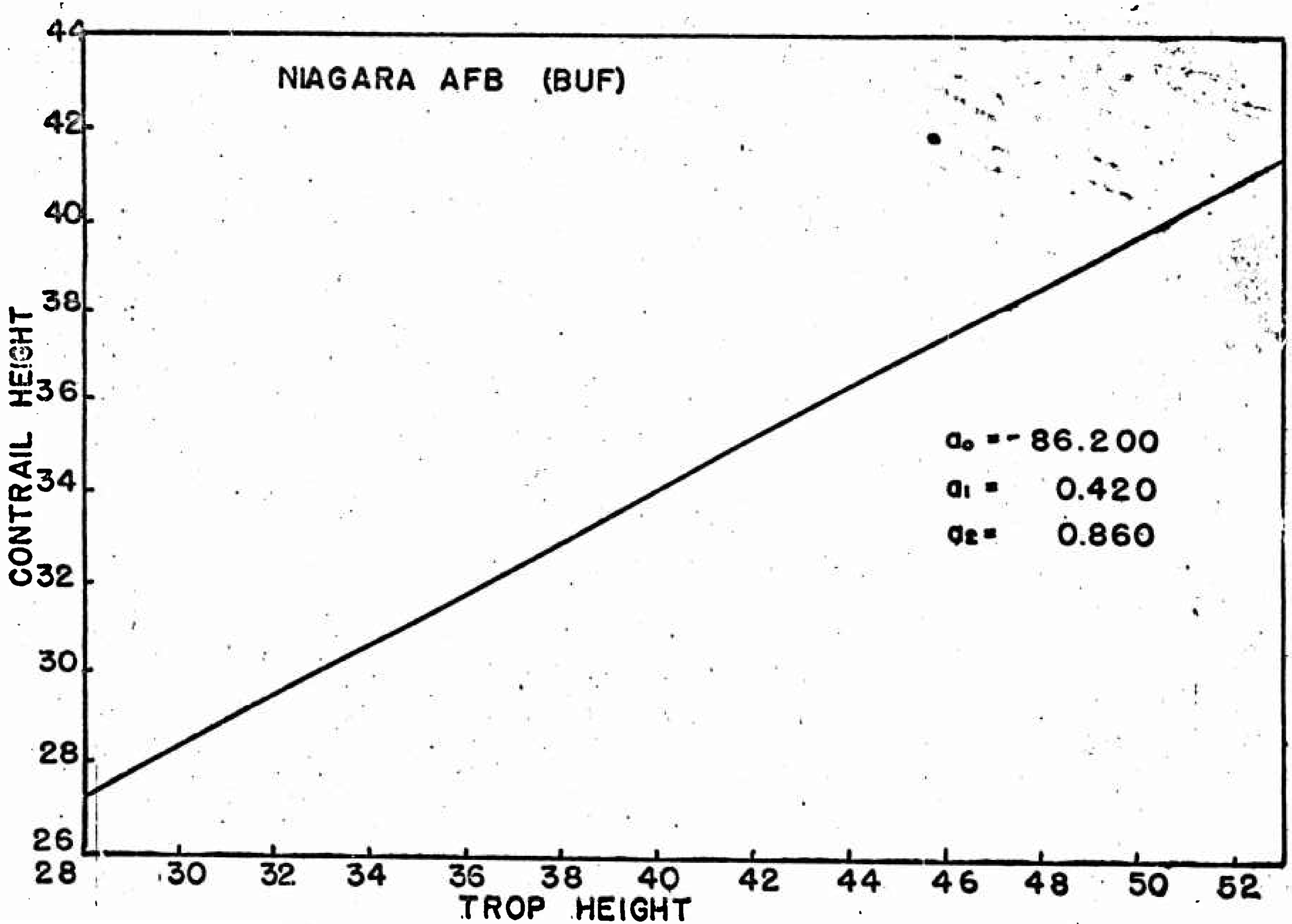


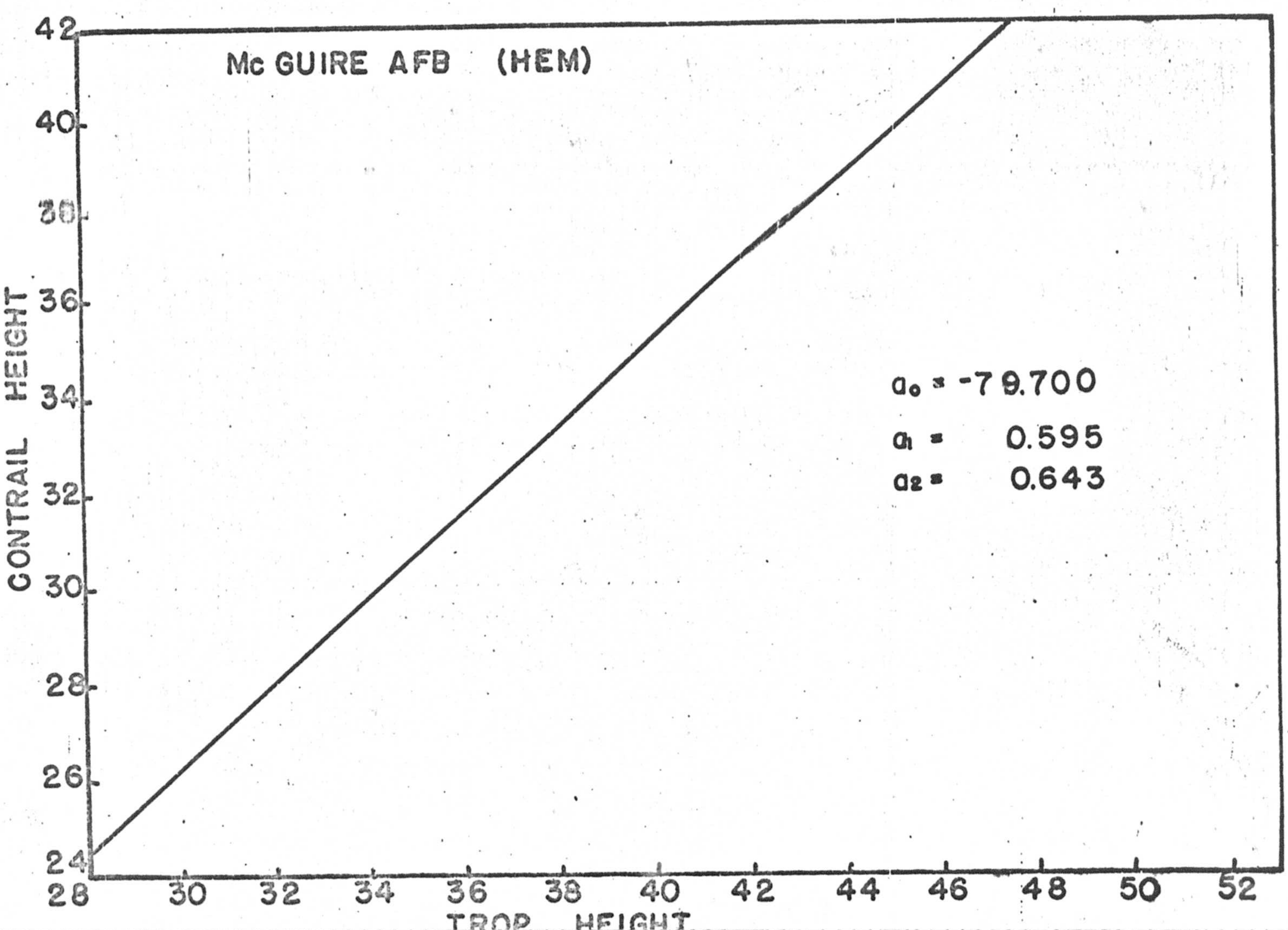
GRIFFITHS AFB (RME)
a_0 = -95.084
a_1 = 0.491
a_2 = 0.816
CONTRAIL HEIGHT
TROP HEIGHT
44
42
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
44
46
48
50
52

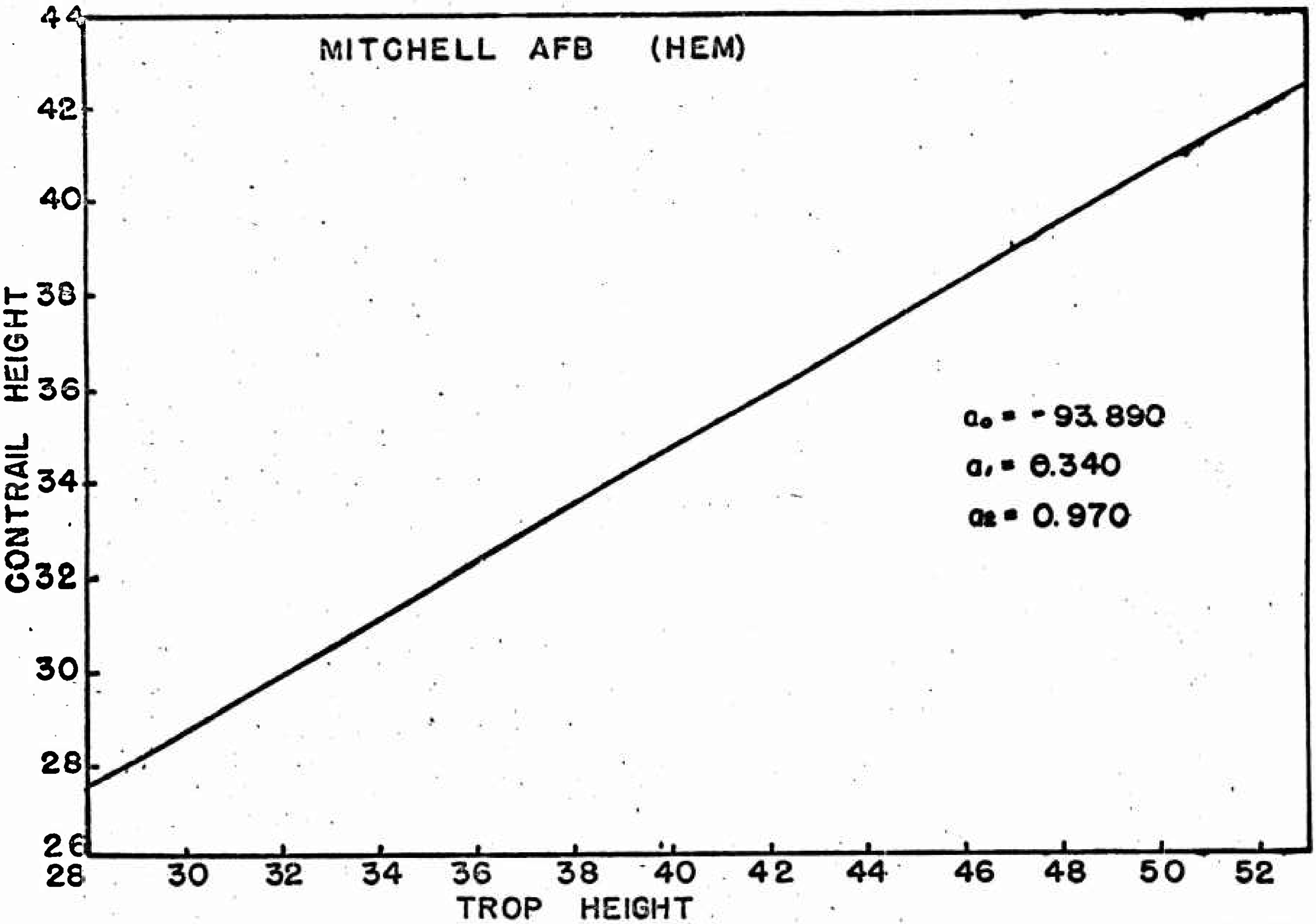

LANGLEY AFB (NGU)
a_0 = -50.000
a_1 = 0.416
a_2 = 0.764
CONTRAIL HEIGHT
TROP HEIGHT
44
42
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
44
46
48
50
52

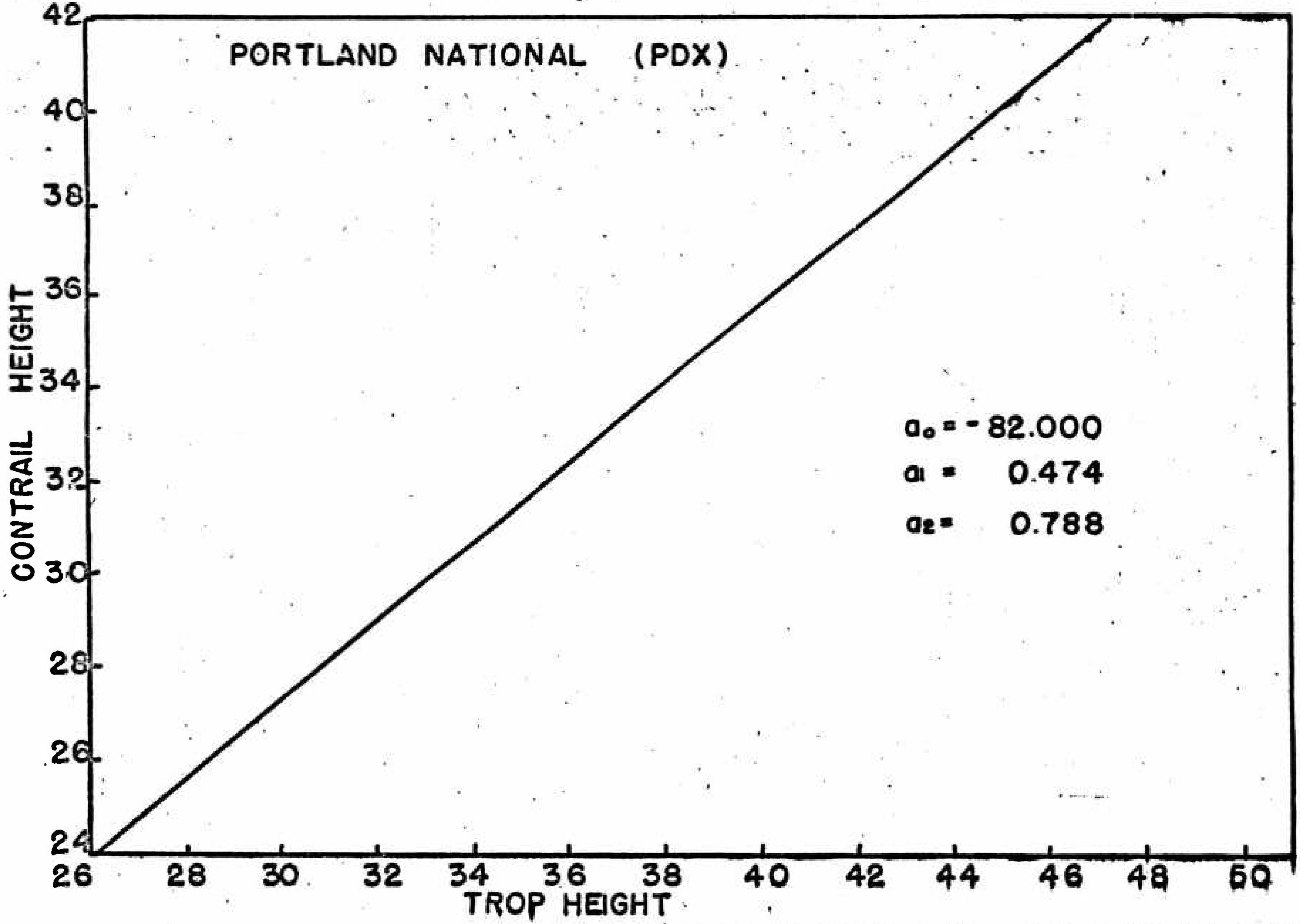


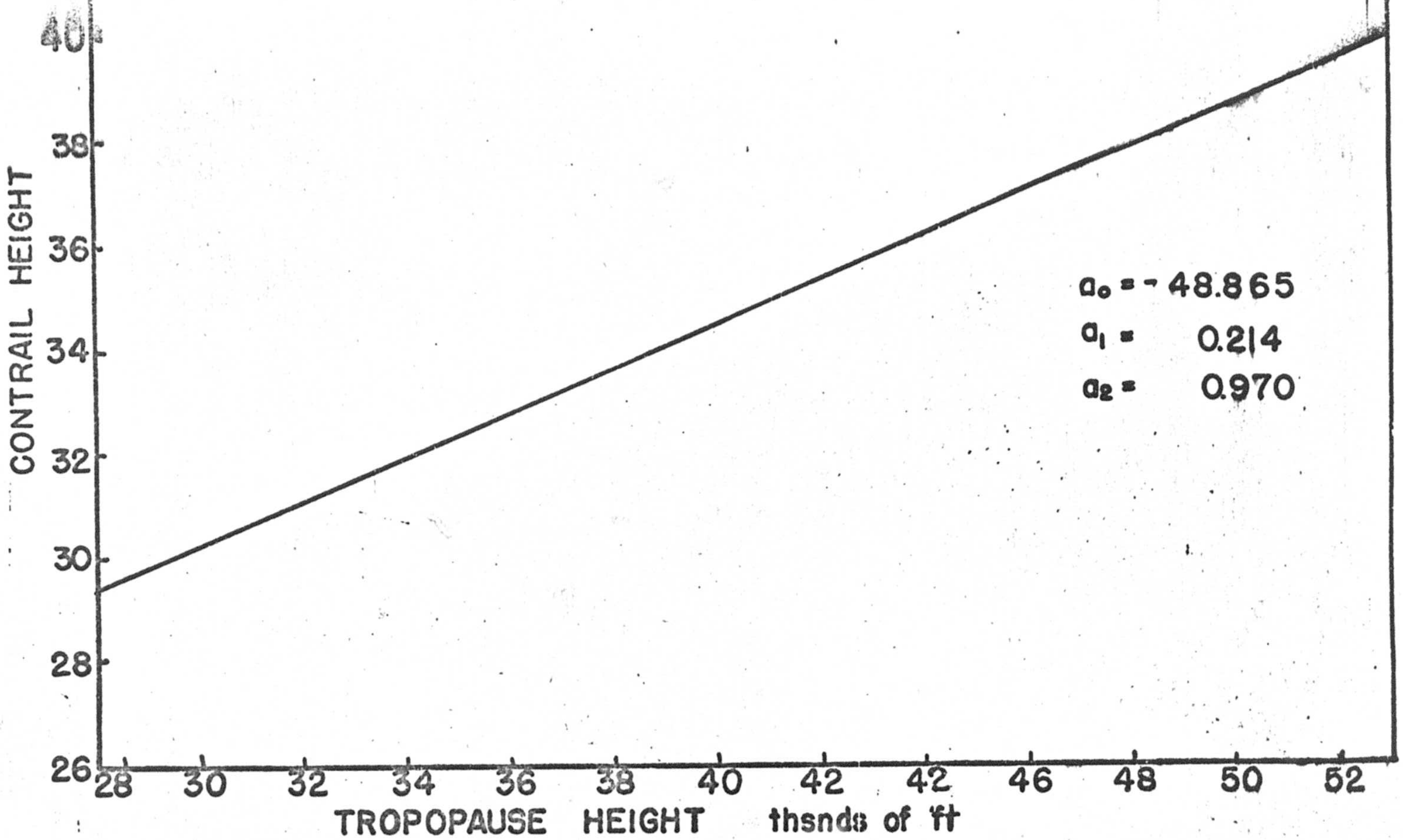
KIRKLAND AFB (AFB)
a₀ =+ 52.000
a₁ = 0.490
a₂ = 0.380
CONTRAIL HEIGHT
TROP- HEIGHT
46
44
42
40
38
36
34
32
30
28
30
32
34
36
38
40
42
44
46
48
50
52
54


NIAGARA AFB (BUF)
a₀ = -86.200
a₁ = 0.420
a₂ = 0.860
CONTRAIL HEIGHT
TROP HEIGHT
26
28
30
32
34
36
38
40
42
44
46
48
50
52

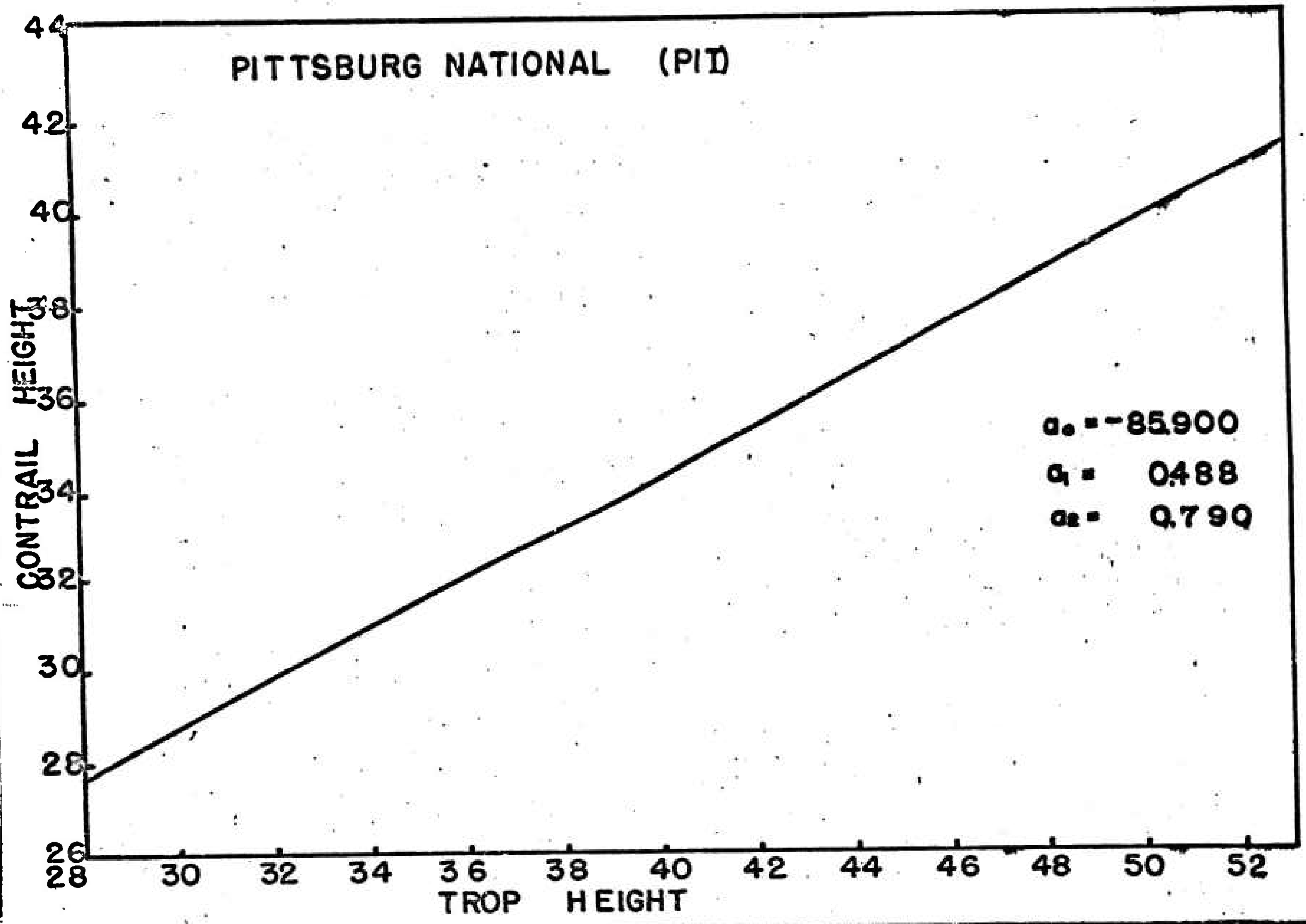

Mc GUIRE AFB (HEM)
a₀ = -79.700
a₁ = 0.595
a₂ = 0.643
CONTRAIL HEIGHT
TROP HEIGHT
42
40
38
36
34
32
30
28
26
24
28
30
32
34
36
38
40
42
44
46
48
50
52


MITCHELL AFB (HEM)
$a_0 = -93.890$
$a_1 = 0.340$
$a_2 = 0.970$
CONTRAIL HEIGHT
44
42
40
38
36
34
32
30
28
26
TROP HEIGHT
28
30
32
34
36
38
40
42
44
46
48
50
52


PORTLAND NATIONAL (PDX)
a_0 = -82.000
a_1 = 0.474
a_2 = 0.788
CONTRAIL HEIGHT
TROP HEIGHT
42
40
38
36
34
32
30
28
26
24
26
28
30
32
34
36
38
40
42
44
46
48
50



CONTRAIL HEIGHT
40
38
36
34
32
30
28
26
28
30
32
34
36
38
40
42
42
46
48
50
52
TROPOPAUSE HEIGHT thsnds of ft
a₀ = - 48.865
a₁ = 0.214
a₂ = 0.970


PITTSBURG NATIONAL (PIT)
a0 = -85.900
a1 = 0.488
a2 = 0.790
CONTRAIL HEIGHT
TROP HEIGHT
26
28
30
32
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38
40
42
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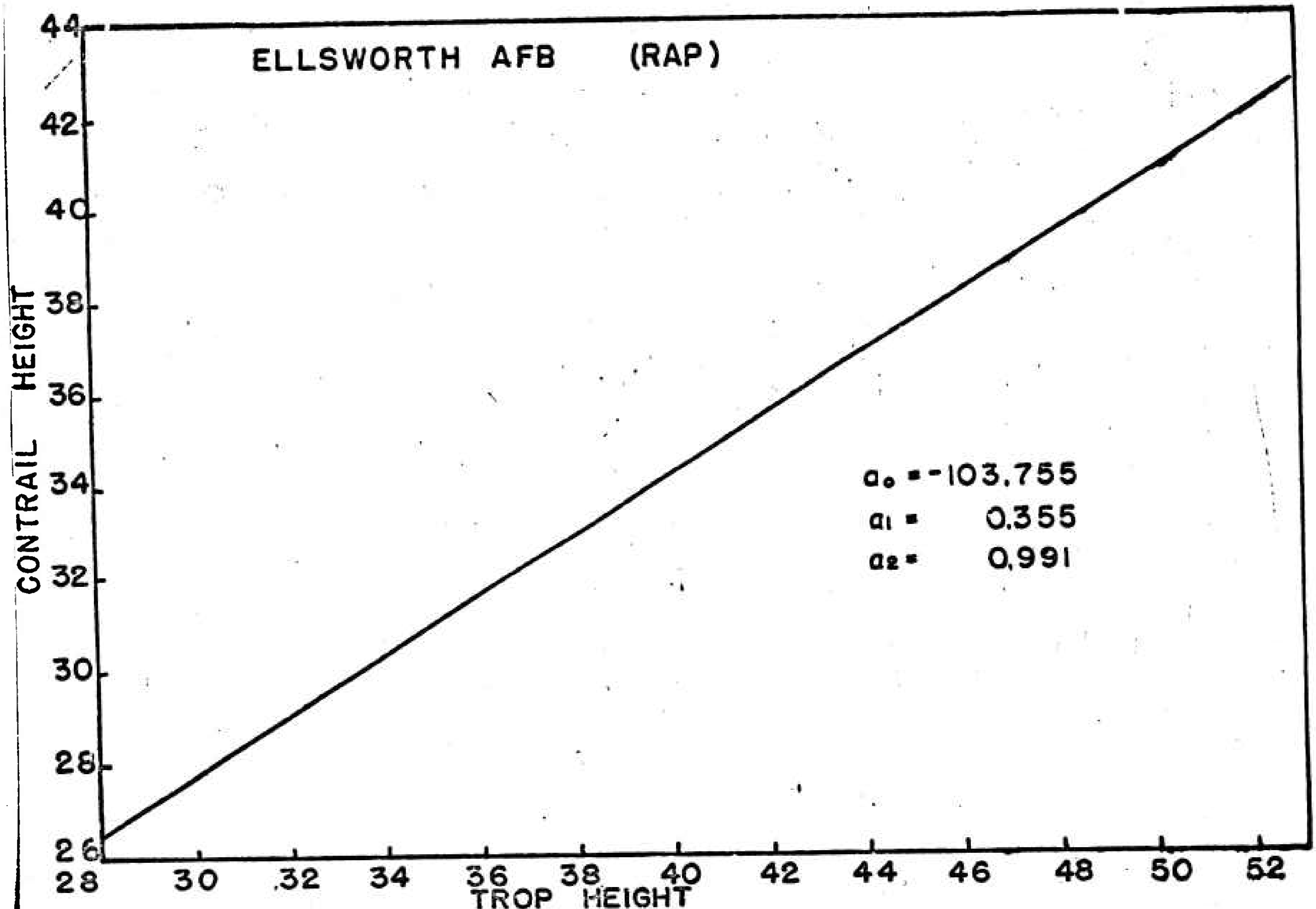
46
48
50
52


ELLSWORTH AFB (RAP)
a_0 = -103.755
a_1 = 0.355
a_2 = 0.991
CONTRAIL HEIGHT
TROP HEIGHT
44
42
40
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